
BRANKO GRÜNBAUM, University of Washington, Seattle, WA 98195-4350, USA

Enumeration of square-faced isogonal infinite polyhedra

The three well-known Coxeter–Petrie polyhedra are examples of regular infinite polyhedra with convex faces. This means that the symmetry group of each acts transitively on its flags, which implies that all faces are congruent regular polygons and that all vertices are equivalent under symmetries of the polyhedron. In the finite case the implication can be reversed resulting in the five Platonic solids. Not so in the infinite case. Adding to previously known results for the infinite case, obtained by various authors, we completely enumerate the infinite polyhedra with square faces, with adjacent faces either coplanar or perpendicular, and with all vertices equivalent under symmetries. There are precisely 15 such polyhedra in which each vertex is incident with five squares, and 6 in which each is incident with six. These enumerations are accomplished via a combination of combinatorial and geometric calculations, both manual and computerized, along with computer graphics visualizations of the polyhedra. The method can be used in more general situations as well.

The work is a collaboration of Steven Gillispie and the presenter, both at the University of Washington.