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Michael coverings are comprehensive

The motivational example for the comprehension scheme (Lawvere '68) came from proof theory. An example with categories as types (Gray '69, Street–Walters '73) exhibited comprehension as the familiar Grothendieck construction of a discrete opfibration associated with a covariant functor $F\colon B \to$ Sets on a small category B.

We introduce the setting of an "extensive 2-doctrine" (E2D) in which to state the comprehension scheme in a 2-categorical setting. This involves a 2-category T "of types" and, for each object X of T, a category $E(X)$ of "extensive quantities of type X" with a terminal object 1_X, and a "pushforward operation" $E(f)\colon E(Y) \to E(X)$ for each 1-cell $f\colon Y \to X$ in T. For each object X of T, we have a 2-functor $B_X\colon (T,X) \to E(X)$ that assigns, to each 1-cell $f\colon Y \to X$, the extensive quantity of type X given by $E(f)(1_Y)$. We say that the E2D satisfies the comprehension scheme if for each X, the 2-functor B_X has a fully faithful right 2-adjoint $\{-\}_B\colon E(X) \to (T,X)$, called comprehension. A 1-cell $f\colon Y \to X$ is called E-dense if the canonical map $E(f)(1_Y) \to 1_X$ is an isomorphism, and it is called an E-covering if the unit $f \to \{B_X(f)\}_X$ is an isomorphism. It follows that every 1-cell $f\colon Y \to X$ admits a unique (up to iso) factorization into an E-dense 1-cell $Y \to Z$, followed by an E-covering 1-cell $Z \to X$. This is called the "E-comprehensive factorization" of f.

The purpose of this talk is:

(1) to remark that the (pure, Fox complete spread) factorization (Bunge–Funk '96) is indeed comprehensive for an E2D with T the 2-category of locally connected (Grothendieck) toposes and $E(X)$ the category of Lawvere distributions on X, and

(2) to prove that the (hyperpure, Michael Complete spread) factorization (Bunge–Funk 2005) is comprehensive for an E2D with T the 2-category of all (Grothendieck) toposes and $E(X)$ a category of what we call "0-distributions", or distributions with values in 0-dimensional locales.

This is joint work with J. Funk.