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The category of Mackey spaces is $*$-autonomous

A standard theorem says that any locally convex topological vector space has a finer topology, its *Mackey topology* with the same set of continuous linear functionals and that is the finest possible topology with that property. If E and F are two such spaces, topologize the space $\mathrm{Hom}(E, F)$ of continuous linear transformations $E \to F$ with the weak topology induced by the algebraic tensor product $E \otimes F'$ and then let $[E, F]$ denote the associated Mackey topology. Let F^* denote the dual F' topologized by the Mackey topology on the weak dual and let $E \otimes F = [E, F^*]^*$ (whose underlying vector space is the algebraic tensor product). Then for any Mackey spaces E, F, and G,

1. $[E \otimes F, G] \cong [E, [F, G]]$
2. $E \cong E^{**}$
3. $[E, F] \cong (E \otimes F^*)^*$

which is summarized by saying that the category of Mackey spaces and continuous linear transformations is $*$-autonomous.

This category is equivalent to the category of weakly topologized locally convex topological vector spaces (which have the coarsest possible topology for their set of continuous linear functionals) which is therefore also $*$-autonomous. They are also equivalent to the chu category of vector spaces (which will be explained).