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Semigroups and toposes

We shall present a strictly semigroup description of the classifying topos $\mathcal{B}(G)$ [?] of an inverse semigroup G. *A left $*$-semigroup* is a semigroup S together with an assignment $s \mapsto s^*$ satisfying:

(i) $(s^*)^* = s$,

(ii) $ss^*s = s$, and

(iii) $(s^*st)^* = (st)^*s$,

for all $s, t \in S$. A morphism of left $*$-semigroups is a function $h\colon S \to T$ such that

(i) $h(s^*) = h(s)^*$, and

(ii) $h(st) = h(s)h(s^*st)$.

Such a morphism h is said to be *étale* if every equation $t = h(f)t$ in T, where f is a strong idempotent ($f = f^*f$) of S, can be lifted uniquely to an equation $s = fs$ in S, meaning $h(s) = t$.

Proposition 1 *$\mathcal{B}(G)$ is equivalent to the category of étale morphisms of left $*$-semigroups over the inverse semigroup G.*

We shall also present a strictly topos description of E-unitary inverse semigroups [?]. A *$\neg\neg$-separated* object of a topos is one that is separated for the $\neg\neg$-topology in the topos [?]. (F is $\neg\neg$-separated iff the diagonal subobject $F \rightarrowtail F \times F$ is equal to its double negation.)

Proposition 2 *An inverse semigroup G is E-unitary iff the object $d\colon G \to E$ of $\mathcal{B}(G)$ is $\neg\neg$-separated, where $E =$ idempotents of G, and $d(t) = t^*t$.*

References

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