Invariant Theory and Differential Geometry
La théorie des invariants et la géométrie différentielle
(Org: **Ray McLenaghan** (Waterloo) and/et **Roman Smirnov** (Dalhousie))

STEPHEN ANCO, Brock University, St. Catharines, ON
Bi-Hamiltonian operators in Lie group geometry and wave maps

In this talk I will show that the recent geometric derivation of bi-Hamiltonian operators for arclength-preserving flows of curves on constant curvature Riemannian manifolds (due to Sanders and Wang, and Mari Beffa) extends to semisimple Lie group manifolds. In particular the bi-Hamiltonian operators are found to be directly encoded in the Cartan structure equations of a left-invariant moving frame associated with a curve and its flow. These operators lead to a hierarchy of commuting flows and conservation laws generated by a recursion operator which has the form of a "square-root" of the corresponding operator known for constant curvature Riemannian manifolds. As one main result, the -1 flow in this hierarchy is shown to be a wave map equation (*i.e.*, nonlinear sigma model) with the Lie group as the target space.

IAN ANDERSON, Utah State University, Logan, Utah
Symbolic Analysis of Lie's Theorem

A fundamental theorem in Lie theory is the classical result (due to Lie himself) which asserts that for each finite dimensional, real Lie algebra g there exists a (local) Lie group G whose associated Lie algebra is the given algebra g. In this talk I will discuss the symbolic implementation of this theorem.

Surprisingly, the standard proof of this theorem (due to Cartan) does not translate into a very useful symbolic algorithm. I will explain why and then give another proof of Lie's theorem which is computationally effective. Applications to invariant theory will be presented.

CLAUDIA CHANU, Università di Torino, via Carlo Alberto 10, 10123 Torino, Italy
Conformal Killing tensors and fixed energy R-separation for the Schroedinger equation

A general geometric framework for the separation of variables in a null PDE of second (or higher) order is presented. The method is applied to the case of the R-separation of the Schrödinger equation with a fixed value of the energy. An intrinsic characterization of the fixed energy R-separation involving conformal Killing tensors is shown.

This is joint work with Giovanni Rastelli.

ALAN COLEY, Dalhousie Univ
Spacetimes with vanishing curvature invariants

All four-dimensional Lorentzian spacetimes with vanishing scalar invariants constructed from the Riemann tensor and its covariant derivatives (VSI spacetimes) are determined. A subclass of the Kundt spacetimes results and the corresponding VSI metrics can be displayed in local coordinates. Some potential applications of VSI spacetimes are discussed. The algebraic classification of the Weyl tensor in higher dimensional Lorentzian manifolds is then described, and higher dimensional VSI spacetimes are discussed.

ROBIN DEELEY, University of Victoria
Invariant Classification of Killing Tensors on the Sphere

In recent years, a method for classifying the orthogonally separable coordinate systems for the Hamilton–Jacobi equation has been developed. This method uses invariants of the vector space of Killing tensors under the action of the isometry group. It has been applied to spaces of constant curvature, including E^2, E^3, M^2 and S^2. This talk focuses on the invariant classification for S^2, along with an alternative classification based on an eigenvalue approach. In addition, if time permits, results for S^3 and S^n will also be discussed.

This is joint work with Ray McLenaghan and Roman Smirnov.

LUCA DEGIOVANNI, Mathematics Departement, University of Torino, via Carlo Alberto 10, 10123 Torino, Italy
Classification of Killing tensor on flat 2-manifolds

An alternative way to obtain the well known classification of Killing tensors and separable coordinates systems, on Euclidean and Minkowski planes, is given. The classification is obtained considering the whole class of transformation that preserve the type of coordinates associated to a given Killing tensor. On flat 2-manifolds, the infinitesimal generator of these transformations form an integrable distribution with rank, in a generic point, equal to the dimension of the vector space of Killing tensors. Thus the integral surfaces of the distribution can be found just looking for the loci where the rank decreases. The process is purely algebraic.

MICHAEL EASTWOOD, University of Adelaide, South Australia 5005
Projective Invariance and Killing Fields

The infinitesimal symmetries on a Riemannian manifold satisfy the Killing equation: $\nabla_{aV_b} = 0$. This equation sees only the projective geometry of the underlying metric, *i.e.*, the geometry of the unparameterised geodesics. I shall explain what this means, its consequences, and how this observation generalises to other Killing equations.

RYAD GHANAM, University of Pittsburgh at Greensburg
Representations for low-dimensional Lie algebras

In this talk we will report on progress in the problem of finding linear representations for low-dimensional real Lie algebras. For each Lie algebra g of dimension less than or equal to 6, we will give a matrix Lie group whose Lie algebra is the given algebra in the list. We will also give a representation of the Lie algebra in terms of vector fields.

SIGBJORN HERVIK, Dalhousie University, Halifax, NS
Spacetimes with Constant Scalar Invariants

In this talk we will discuss spacetimes with constant scalar invariants. There are many examples of such spacetimes, among them spacetimes with vanishing curvature invariants and homogeneous spaces. A certain class of spacetimes to which all known examples belong, as well as their mathematical and physical properties, will be discussed.

JOSHUA HORWOOD, University of Cambridge, Department of Applied Mathematics and Theoretical Physics, Wilberforce Road, Cambridge CB3 0WA, United Kingdom
Classification of orthogonal coordinate webs in three-dimensional Minkowski space

The use of isometry group invariants to classify orthogonally separable Hamiltonian systems and their associated orthogonal coordinate webs in spaces of constant curvature has been remarkably successful on the Euclidean and Minkowski planes. Recently, Horwood, McLenaghan and Smirnov derived an invariant-based classification for the eleven orthogonal coordinate webs in three-dimensional Euclidean space. In this talk, I will focus on a substantially harder problem, namely three-dimensional Minkowski space, for which there are fifty distinct coordinate systems which permit orthogonal separation of the associated Hamilton-Jacobi and Helmholtz equations. I will outline an invariant classification scheme for the corresponding thirty-eight orthogonal coordinate webs, emphasizing not only the role of the group invariants in its development, but also the importance of group covariants, reduced invariants and conformal symmetries.

NIKY KAMRAN, McGill University
Null surfaces and contact geometry

We will start with a survey of the null surface formulation of the Einstein field equations of gravitation, which has been developed over many years by Newman and his collaborators. We will then show how this makes it possible to rediscover the isomorphism between three-dimensional conformal Lorentzian geometry and and the contact geometry of third order ordinary differential equations, first brought to light by Cartan and Chern. We will also show that four-dimensional conformal Lorentzian geometry corresponds to the contact geometry of a class of overdetermined system of second-order pdes in two independent variables.

IRINA KOGAN, North Carolina State University
Rational and Algebraic Invariants and the Moving Frame Method

We consider rational actions of the connected algebraic groups on an affine space, and provide algorithms for constructing finite generating sets of rational and algebraic invariants, together with the algorithms for rewriting any rational invariant in terms of the generators. The construction of algebraic invariants, we propose, can be seen as an algebraic counterpart of the Fels and Olver moving frame construction for local smooth invariants on a differential manifold. In particular, we provide an algebraic equivalent for the notions of cross-section and invariantization. The algebraic formulation reduces all algorithms to Groebner bases computations, and can be easily implemented in any computer-algebra system. A generating set of rational invariants is obtained as a side product of our algorithm for constructing a generating set of algebraic invariants.

This is a joint work with E. Hubert, INRIA, France.

WILLARD MILLER, JR., University of Minnesota, Minneapolis, Minnesota
Second-order superintegrable systems

A Schrödinger operator with potential on a Riemannian space is 2nd-order superintegrable if there are $2n-1$ (classically) functionally independent 2nd order symmetry operators. (The $2n-1$ is the maximum possible number of such symmetries.) These completely integrable Hamiltonian systems are of special interest because they are multi-integrable, even multiseparable, *i.e.*, variables separate in several coordinate systems, and the systems are explicitly solvable in terms of special functions.

We first give examples of superintegrable systems and then we present very recent results giving the general structure of superintegrable systems in all 2D, and 3D conformally flat spaces, and a complete list of such spaces and potentials in 2D. The results reported here were obtained in collaboration with E. G. Kalnins, G. S. Pogosyan and J. Kress.

ROBERT MILSON, Dalhousie University
Killing tensors as irreducible representations of the general linear group

We show that the vector space of fixed valence Killing tensors on a space of constant curvature is naturally isomorphic to a certain irreducible representation of the general linear group. The isomorphism is equivariant in the sense that the natural action of the isometry group corresponds to the restriction of the linear action to the appropriate subgroup. As an application, we deduce the Delong–Takeuchi–Thompson formula on the dimension of the vector space of Killing tensors from the classical Weyl dimension formula.

ANATOLY NIKITIN, Institute of Mathematics, Kiev, Ukraine
On the Galilean vector covariants

I will present a complete list of the Galilean vector covariants and describe the Galilei invariant wave equations for vector and spinor fields.

PETER OLVER, University of Minnesota
Lie pseudo-groups

A theory of moving frames is developed for Lie pseudo-groups, leading to new, explicit computational algorithms for determining their structure and the structure of their differential invariant algebra. The talk will focus on symmetry (pseudo-)groups of differential equations and variational problems.

DENNIS THE, McGill University, Dept. of Math & Stats, 805 Sherbrooke St. West, Montreal, QC H3A 2K6
Symmetries, conservation laws, and cohomology of Maxwell's equations using potentials

We discuss the symmetry and conservation law structure for the free-space Maxwell's equations in Minkowski space and two of its potential systems:

(1) the standard Lagrangian potential system using $F = dA$, and

(2) a natural potential system obtained by introducing joint covariant vector potentials on both F and its (Hodge) dual $*F$.

In the absence of gauge constraints, the local symmetry, adjoint-symmetry and conservation law structure of these systems follows as a consequence of their local 1-form and 2-form cohomology, together with a general theorem describing how local symmetries of a potential system with gauge freedom project to local symmetries of the original system.

With Lorentz gauge imposed, the standard potential system is well known to inherit the Killing symmetries of Maxwell's equations, but not the inversion (conformal) symmetries. In contrast, the joint potential system with Lorentz gauges imposed admits inversion-type symmetries, as we show by a classification of first-order symmetries (of a certain geometric form) for this system. Finally, we derive new nonlocal classes of symmetries and conservation laws of Maxwell's equations as a result of this classification.

This talk is based on joint work with Stephen Anco.

THOMAS WOLF, Brock University, St. Catharines, ON
Integrable Quadratic Hamiltonians on $\mathrm{so}(4)$ *and* $\mathrm{so}(3,1)$

In the talk a special class of quadratic Hamiltonians on $\mathrm{so}(4)$ and $\mathrm{so}(3,1)$ is discussed. Results include a Hamiltonian with a 6-th degree first integral and a superintegrable Hamiltonian together with inhomogeneous generalizations.

This work was done in collaboration with Vladimir Sokolov.

If time permits, a vector formalism will be introduced which allows us to reach the same results more efficiently.

ISMET YURDUSEN, Centre de Recherches Mathématiques (CRM), Université de Montréal P.O. Box 6128, Centre-ville Station, Montréal, Québec H3C 3J7
Prolongation Structure and Integrability of the coupled KdV-mKdV system

Recently, Kersten and Krasil'shchik constructed the recursion operator for a coupled KdV-mKdV system, which arises as the classical part of one of superextensions of the KdV equation. In this work, we study the integrability of this system using the Painlevé test. Then, we use the Dodd–Fordy algorithm for the Wahlquist–Estabrook prolongation technique in order to obtain the Lax pair. We find a 3×3 matrix spectral problem for the Kersten and Krasil'shchik system. We also show that the Lax pair obtained is a true Lax pair since the spectral parameter cannot be removed by a gauge transformation, as can be proven by a gauge-invariant technique.

This is a joint work with Ayse Karasu and Sergei Yu. Sakovich.