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Thurston laminations of the unit disk, equivalence relations, and polynomial Julia sets

In a widely circulated preprint (1984) William Thurston introduced the notion of a (geodesic) lamination of the unit disk. Laminations are combinatorial/geometric/topological objects used to study Julia sets of polynomials in analytic complex dynamics. A lamination of the unit disk is a closed collection of chords of the disk that do not cross each other (they may touch at endpoints). Consider the power map $f(z) = z^d$, $d > 1$, on the unit circle; extend f linearly to the lamination (the chords). A chord is critical if its endpoints map to one point. A lamination is invariant if the collection maps to itself forward and backward, with d-many disjoint pre-images of each chord backward, and f extends linearly to a positively-oriented confluent map of the disk to itself. The plan is that

(1) a lamination is determined by 'pulling back' a set of critical chords,

(2) the lamination naturally induces an equivalence relation on the unit circle,

(3) the quotient space of the circle under this equivalence relation is a topological Julia set, and

(4) the topological Julia set is dynamically (and topologically) equivalent to an analytic Julia set for some degree d polynomial.

But there are obstructions to the fulfillment of the plan. Thurston completed most of the plan for $d = 2$, but left some questions unanswered. Moreover, fundamental questions remain unanswered for $d > 2$, but recent progress has been made. In particular, one obstruction is that the lamination determined by a collection of critical chords may naturally induce a degenerate equivalence relation, collapsing the circle to a point in the quotient. In this talk, we show how the obstruction arises in degree $d = 2$, and give some insight into degree $d = 3$ and greater. In a subsequent talk at this meeting, D. Childers provides a complete solution to when degeneracy occurs, for degree $d = 2$, in terms of the dynamics of the critical chord, answering an implicit question of Thurston.

This talk is mostly joint work with members of the UAB Laminations Seminar: A. Blokh, L. Oversteegen, D. Childers, G. Brouwer, C. Curry, and P. Eslami.