In a widely circulated preprint (1984) William Thurston introduced the notion of a (geodesic) lamination of the unit disk. Laminations are combinatorial/geometric/topological objects used to study Julia sets of polynomials in analytic complex dynamics. A lamination of the unit disk is a closed collection of chords of the disk that do not cross each other (they may touch at endpoints). Consider the power map \( f(z) = z^d, \ d > 1 \), on the unit circle; extend \( f \) linearly to the lamination (the chords). A chord is critical if its endpoints map to one point. A lamination is invariant if the collection maps to itself forward and backward, with \( d \)-many disjoint pre-images of each chord backward, and \( f \) extends linearly to a positively-oriented confluent map of the disk to itself. The plan is that

1. a lamination is determined by ‘pulling back’ a set of critical chords,
2. the lamination naturally induces an equivalence relation on the unit circle,
3. the quotient space of the circle under this equivalence relation is a topological Julia set, and
4. the topological Julia set is dynamically (and topologically) equivalent to an analytic Julia set for some degree \( d \) polynomial.

But there are obstructions to the fulfillment of the plan. Thurston completed most of the plan for \( d = 2 \), but left some questions unanswered. Moreover, fundamental questions remain unanswered for \( d > 2 \), but recent progress has been made. In particular, one obstruction is that the lamination determined by a collection of critical chords may naturally induce a degenerate equivalence relation, collapsing the circle to a point in the quotient. In this talk, we show how the obstruction arises in degree \( d = 2 \), and give some insight into degree \( d = 3 \) and greater. In a subsequent talk at this meeting, D. Childers provides a complete solution to when degeneracy occurs, for degree \( d = 2 \), in terms of the dynamics of the critical chord, answering an implicit question of Thurston.

This talk is mostly joint work with members of the UAB Laminations Seminar: A. Blokh, L. Oversteegen, D. Childers, G. Brouwer, C. Curry, and P. Eslami.