Applications of selections to the Hahn–Mazurkiewicz Problem

The Hahn–Mazurkiewicz Problem asks for conditions under which a Hausdorff space is the continuous image of a generalized arc. The first characterizations of continuous images of non-metric arcs were given by Bula and Turzanski and by Nikiel. Additional results include those of Mardešić, Treybig, and many others.

In a related study, we herein consider applications of selections (carriers) to the study of images of ordered compacta. In particular, let $X$ be a compact ordered space, $Y$ a Hausdorff space, and let $F(Y)$ denote the family of all nonempty closed subsets of $Y$ with the Vietoris topology. Assuming $G: X \to F(Y)$ is continuous, we consider conditions under which $G$ can be "lifted" to a continuous map of $X$ onto $Y$.

This work relies heavily on work of R. S. Countryman as well as the theory of selections and that of continuous images of ordered compacta.