Suppose that $V$ is a vector space over $\mathbb{Q}$, $\mathbb{R}$ or $\mathbb{C}$, the scalars $\alpha_0, \beta_0, \ldots, \alpha_m, \beta_m$ are such that $\alpha_j \beta_k - \alpha_k \beta_j \neq 0$ whenever $0 \leq j < k \leq m$, $B$ is a Banach space, $f_k : V \to B$ for $0 \leq k \leq m$, $\delta \geq 0$ and

$$\left\| \sum_{k=0}^{m} f_k (\alpha_k x + \beta_k y) \right\| \leq \delta \quad \text{for all } x, y \in V.$$

Then, for each $k = 0, 1, \ldots, m$ there exists $c_k \in B$ and a "generalized" polynomial function $p_k : V \to B$ of "degree" at most $m - 1$, such that

$$\| f_k (x) - c_k - p_k (x) \| \leq 2^{m+1} \delta \quad \text{for all } x \in V$$

and

$$\sum_{k=0}^{m} p_k (\alpha_k x + \beta_k y) = 0 \quad \text{for all } x, y \in V.$$

Moreover, if $V = \mathbb{R}^n$, $B = \mathbb{R}$ or $\mathbb{C}$ and, for some $j$, $f_j$ is bounded on a set of positive Lebesgue measure, then every $p_k$ is a genuine polynomial function.