Any finite or infinite Coxeter group $G$ with string diagram is the automorphism group of an abstract regular polytope. When $G$ is crystallographic, its standard real representation is easily reduced modulo an odd prime $p$, thus giving a finite representation in some finite orthogonal space $V$ over the field with $p$ elements. The finite group need not be polytopal; and whether or not it is depends in an intricate way on the geometry of $V$. The talk presents recent work with Barry Monson, in which we describe this construction in considerable generality and study in depth the interplay between the geometric properties of the polytope (if it exists) and the algebraic structure of the overlying finite orthogonal group. The rank 4 case has been worked out in considerable detail.