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Voronoi's Conjecture on Parallelotopes

A parallelotope is a polytope with the following, very restrictive, combinatorial property: a selection of translates of the polytope fit together facet-to-facet to tile space. It is easy to show that the centers of the parallelotopes of such a tiling form a lattice. On the other hand, starting with a geometric lattice, the Voronoi polytope is determined by the Euclidean metric: the Voronoi polytope for a particular lattice point is the set of points as close to that lattice point as to any other. The Voronoi polytopes for all lattice points fit together facet-to-facet to tile space, so a Voronoi polytope is a special type of parallelotope. Voronoi conjectured in 1909 that these two tilings, definded is such different ways, are in fact equivalent; Voronoi conjectured each parallelotope tiling is affinely equivalent to a Voronoi tiling.

I will report on the recent work of Andrei Ordine who established that the Voronoi Conjecture holds for a new special case—for the case where the parallelotope is 3-irreducible, a condition that will be defined during the course of my lecture. I will also review the status of the Voronoi Conjecture, describing the various cases for which the Conjecture has been proven to hold, and discuss prospects for further developments.