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On Smoothness and Gorensteinness of Schubert varieties

The study of singularities of Schubert varieties in the flag manifold involves interesting interplay between algebraic geometry, representation theory and combinatorics.

Although all Schubert varieties are Cohen–Macaulay, few are smooth. An explicit combinatorial characterization of the smooth ones was given by Lakshmibai and Sandhya (1990). The singular locus of an arbitrary Schubert variety was determined around 2001 by several authors.

Gorensteinness is a measurement of the “pathology” of the singularities of an algebraic variety; it logically sits between smoothness and Cohen–Macaulayness. We explicitly characterize which Schubert varieties are Gorenstein, analogous to Lakshmibai and Sandhya’s theorem. Here is the geometric interpretation: a Schubert variety is Gorenstein if and only if it is Gorenstein at the generic points of the singular locus. We also compute the canonical sheaf of a Gorenstein Schubert variety as a line bundle in terms of the Borel–Weil construction.

I will discuss the geometric corollaries and questions that arise in this work. This is a joint project with Alexander Woo.