MICHEL RIGO, University of Liege, Department of Mathematics, Grande Trave 12 (B37), B-4000 Liege, Belgium
Abstract numeration systems, additive functions and automatic sequences

Abstract numeration systems are a natural generalization of positional numeration systems whose set of representations of all the integers is a regular language. They were introduced five years ago by P. Lecomte and myself [3] and are defined in the following way. Consider any infinite regular language L over a totally ordered alphabet $(A, <)$. An abstract numeration system is thus simply given by the triple $S = (L, A, <)$. The words of L can be enumerated with respect to the genealogical ordering induced by the total ordering of A. We therefore have a one-to-one correspondence between the set of nonnegative integers and the language L. We say that the $(n+1)$-st word of L is the *S-representation* of the integer n.

In this general setting, various notions arising in the study of "classical" numeration systems (p-ary system or systems built over a strictly increasing sequence of integers like the Fibonacci system) can be generalized and may give rise to new phenomena. In particular, if we consider a finite automaton with output fed with the S-representations of the successive integers, we obtain a notion of "generalized" automatic sequence or *S-automatic sequence* [5], [6].

In this talk, I will mainly focus on a problem related to completely additive functions defined over the alphabet A and taking real values, *i.e.*, $f(a_1 \cdots a_k) = f(a_1) + \cdots + f(a_k)$ for any finite word $a_1 \cdots a_k$ over A. With P. Grabner, we have studied, in the framework of abstract numeration systems, as a first step the asymptotic behaviour of the summatory function of additive functions [1] and as a second step the distribution of such functions [2]. The obtained results can naturally be applied to study the frequency of a symbol appearing in a generalized automatic sequence (assuming that the frequency exists) [4].

References

[1] P. Grabner, M. Rigo, *Additive functions with respect to numeration systems on regular languages.* Monatsh. Math. **139**(2003), 205–219.

[2] ———, *Distribution of additive functions with respect to numeration systems on regular languages*. Submitted.

[3] P. B. A. Lecomte and M. Rigo, *Numerations systems on a regular language.* Theory of Comput. Syst. **34**(2001), 27–44.

[4] S. Nicolay and M. Rigo, *About the density of generalized automatic sequences.* Submitted.

[5] M. Rigo, *Generalization of automatic sequences for numeration systems on a regular language*. Theoret. Comput. Sci. **244**(2000), 271–281.

[6] M. Rigo and A. Maes, *More on generalized automatic sequences*. J. Autom. Lang. Comb. **7**(2002), 351–376.

Various preprints are available on my homepage `http://www.discmath.ulg.ac.be`.