Mathematics of Actuarial Finance
Mathématiques financières actuarielles

(Org: **Tom Salisbury** (York; Fields))

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Minimizing the Probability of Lifetime Ruin under Borrowing Constraints

We determine the optimal investment strategy of an individual who targets a given rate of consumption and who seeks to minimize the probability of going bankrupt before she dies, also known as *lifetime ruin*. We impose two types of borrowing constraints: First, we do not allow the individual to borrow money to invest in the risky asset nor to sell the risky asset short. However, the latter is not a real restriction because in the unconstrained case, the individual does not sell the risky asset short. Second, we allow the individual to borrow money but only at a rate that is higher than the rate earned on the riskless asset.

We consider two forms of the consumption function:

(1) The individual consumes at a constant (real) dollar rate, and

(2) the individual consumes a constant proportion of her wealth.

The first is arguably more realistic, but the second is closely connected with Merton's model of optimal consumption and investment under power utility. We demonstrate that connection in this paper, as well as include numerical examples to illustrate our results.

This is a joint work with Virginia R. Young.

PHELIM BOYLE, School of Accountancy, University of Waterloo

Modeling long term embedded options: Actuarial finance in action

Contemporary life insurance products often include embedded options. These options are markedly different from standard financial options in that they are long term and their value depends on financial variables and policyholder behaviour. They are difficult to price and their risk management is challenging. Not surprisingly, consumers find it hard to evaluate their worth. In this introductory talk we will discuss the modeling issues involved in the pricing, valuation and risk management of these options. Their valuation requires not only an analysis of the relevant financial variables but also an analysis of how the actuarial assumptions (such as lapse behaviour) affect the risk. Each of these presents a challenge, but the problem is exacerbated by an interaction of these factors. Policyholder behaviour is influenced to some extent by economic conditions but we do not have good theories on exactly how policyholders will behave and there is a paucity of published experience data.

JOSÉ GARRIDO, Department of Mathematics and Statistics, Concordia University

Properties of distortion risk measures

The current actuarial-financial literature does not reach a consensus on which risk measures should be used in practice. Our objective is to give at least a partial solution to this problem. In this paper we study properties that a risk measure must satisfy in order to avoid some of the "inconsistencies" observed with popular measures like VaR.

We review the reasons why certain risk measures, like Conditional Value at Risk (CVaR) can, in some cases, lead to erroneous decisions. Some properties are proposed so that risk measures can avoid such inconsistencies. This leads to the definition of two new families of risk measures: *complete* measures and *adapted* measures.

In particular, we study the set of risk measures that are based on distortion functions and characterize the completeness and adaptive properties of these, in terms of the derivative of the distortion function that defines the risk measure.

This is joint work with Alejandro Balbás and Silvia Mayoral (Madrid).

SEBASTIAN JAIMUNGAL, University of Toronto
Catastrophe Options with Stochastic Interest Rates

This talk will focus on the pricing and hedging of catastrophe put options when interest rates are stochastic and losses are generated by a compound Poisson process. The asset price process is modeled through a jump-diffusion process that is correlated to the loss process. We obtain explicit formulæ for the price of the option, and the Greek hedging parameters Delta, Gamma and Rho. Furthermore, numerical experiments are carried out to illustrate the effect that stochastic interest rates and the variance of the loss process have on option prices. Finally, we explore some simulation results to study the effectiveness of a Delta-Gamma-Rho hedging scheme.

This is joint work with Tao Wang.

ALEXANDER MELNIKOV, University of Alberta
Quantile hedging for Actuarial Risk Management

The main goals of the talk are: to investigate how quantile hedging developed in mathematical finance can be applied to equity-linked life insurance; to perform actuarial analysis to illustrate risk management implications for insurance companies; and to examine how the choice of a particular survival model affects assessment of mortality risk. The approach will be illustrated by presenting numerical examples based on appropriate financial and mortality data.

KRISTIN MOORE, University of Michigan, Department of Mathematics, Ann Arbor, MI 48109-1043
Optimal and Nearly-Optimal Strategies for Minimizing the Probability of Ruin in Retirement

The increasing risk of poverty in retirement has been well-documented; this phenomenon is driven by demographic trends, changes in employer-sponsored pension plans, and inadequate private retirement savings. We study the optimal investment strategy for a retiree whose objective is to minimize the probability of lifetime ruin, namely the probability that a fixed consumption strategy will lead to zero wealth while the individual is still alive. We derive a variational inequality that governs the ruin probability and the optimal strategy, and we demonstrate that the problem can be recast as a related optimal stopping problem which yields a free-boundary problem that is more tractable. In the special case of exponential future lifetime, one can solve the free-boundary problem explicitly and recover a concise expression for the optimal asset allocation. For more general mortality, we numerically calculate the ruin probability and optimal strategy and examine how they change as we vary the mortality assumption and parameters of the financial model. In addition, we consider suboptimal strategies that are easier to implement and examine the impact on the ruin probabilities.

This is joint work with Virginia Young.

MANUEL MORALES, Department of Mathematics and Statistics, York University
On the discounted penalty function for the generalized inverse Gaussian process

We will review, from a historical point of view, the use of Lévy processes in ruin theory. We focus on the decomposition for the ruin probability and we argue how its convolution structure is inherited from the Lévy family of processes. We will discuss the notion of discounted penalty function in the framework of Lévy risk processes. The problem of finding expressions for this function in a risk model driven by a Lévy process will be addressed. The particular example using a generalized inverse Gaussian process will be discussed. In this case, integral expressions for the discounted penalty function are available. Actual computation of ruin probabilities, distribution of the time of ruin and joint distribution of the process prior and at the moment of ruin, are carried out for a this example. Finally, forms for the discounted penalty function in more general Lévy risk models will be presented.

This is joint work with Jose Garrido (Concordia).

DAVID PROMISLOW, Dept. of Mathematics & Statistics, York University, Toronto
Pension fund switching

A recent trend in pensions is to allow employees a choice between defined benefit and defined contribution plans. In some cases there are possibilities to switch from one to the other. A particular example is the State of Florida, which in 2002 implemented a scheme whereby employees could choose a self managed DC plan, in place of the usual DB plan, but at any time before

maturity they could elect to switch back to the DB plan, upon payment of the accrued benefit obligation. We analyze the effect of this provision on the various parties.

This is joint work with Moshe Milevsky.

KEN SENG TAN, University of Waterloo, Dept. of Statistics and Actuarial Science

CTE and Capital Allocation under the Skew Elliptical Distributions

In recent years, there has been a growing interest among actuaries and finance experts on adopting the Conditional Tail Expectation (CTE) as a "coherent" risk measure for risk management. More recently CTE has been proposed in the context of capital allocation in which CTE provides a convenient way of determining the capital requirement for individual lines of business among correlated business units. Analytic results are derived in Panjer (2002) in the case of multivariate Normal risks. Landsman and Valdez (2003) extend the results of Panjer (2002) to elliptical distributions. In this paper, we further generalize these results by considering a relatively new class of distributions known as the skew elliptical distributions. These distributions have the desirable properties that they need not be symmetric and there is an additional parameter which regulates the skewness.

This is joint work with Jun Cai.

STEVEN VANDUFFEL, Katholieke Universiteit Leuven, and Fortis Central Risk Management, Belgium

Closed-Form Approximations for Constant Continuous Annuities

For a series of cash flows, the stochastically discounted or compounded value is often a key quantity of interest in finance and actuarial science. Unfortunately, even for the most realistic rate of return models, it may be too difficult to obtain analytic expressions for the risk measures involving this discounted sum. Some recent research has demonstrated that in the case where the return process follows a Brownian motion, the so-called comonotonic approximations usually provide excellent and robust estimates of risk measures associated with discounted sums of cash flows involving log-normal returns.

We will derive analytic approximations for risk measures in case one considers the continuous counterpart of a discounted sum of log-normal returns. Although one may consider the discrete sums as providing a more realistic situation than their continuous counterpart, considering the continuous setting leads to more tractable explicit formulas and may therefore provide further insight necessary to expand the theory and to exploit new ideas for later developments. Moreover, the closed-form approximations we derive in this continuous set-up can then be compared more effectively with some exact results, thereby facilitating a discussion about the accuracy of the approximations. Indeed, in the discrete setting, one must always compare approximations with results from simulation procedures, which always gives room for debate.

Our numerical comparisons reveal that the comonotonic "maximal variance" lower bound approximation provides an excellent fit for several risk measures associated with integrals involving log-normal returns. Similar results to those we derive for continuous annuities can also be obtained in the case of continuously compounding which therefore opens a roadmap for deriving closed-form approximations for the prices of Asian options. Future research will also focus on optimal portfolio slection problems.

This is joint work with Jan Dhaene and Emil Valdez.

JIAN WANG, School of Computer Science, University of Waterloo

Hedging with a Correlated Asset: An Insurance Approach

Hedging a contingent claim with an asset which is not perfectly correlated with the underlying asset results in an imperfect hedge. One example arises in the case of segregated funds. These are guarantees provided by insurance companies on mutual fund investments. In many cases, the underlying asset is a mutual fund managed by the insurer providing the guarantee. As the insurance company cannot take a short position in its own fund, the guarantee would typically be hedged using index futures, which will lead to an imperfect hedge. We price the residual risk from hedging with a correlated asset using an actuarial standard deviation principle in infinitesimal time, which leads to a nonlinear partial differential equation. A fully implicit, monotone discretization method is developed for solving the pricing PDE. This method is shown to converge to the viscosity solution, provided certain grid conditions are satisfied. An algorithm is devised to ensure that these conditions hold. Monte Carlo simulations are used to illustrate features of the profit and loss distribution from hedging a contingent claim with an imperfectly correlated asset.

This is joint work with P. A. Forsyth, K. R. Vetzal, and H. A. Windcliff.

VIRGINIA YOUNG, Department of Mathematics, University of Michigan
Correspondence between Lifetime Minimum Wealth and Utility of Consumption

We establish when the two problems of minimizing a decreasing function of lifetime minimum wealth and of maximizing utility of lifetime consumption result in the same optimal investment strategy. To this end, we equate the two investment strategies and show that if the individual consumes at the same rate in both problems—the consumption rate is a control in the problem of maximizing utility—then the investment strategies are equal only when the consumption function is linear in wealth. It then follows that the corresponding investment strategy is also linear in wealth and the implied utility function exhibits hyperbolic absolute risk aversion.

This is joint work with Erhan Bayraktar.