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Estimates for the Probability Itô Processes Remain Around a Curve, and Applications to Finance

We consider a Brownian Motion $W=(W^i)_{i\in N}$ and an adapted process of dimension n $(X_t)_{t\geq 0}$. We define $\tau_R=\inf\{t: |X_t-x_t|\geq R_t\}$, where x_t, $t\geq 0$ is a deterministic differentiable curve in R^n and $R_t>0$, $t\geq 0$ a ratio that depends on time. Assume that until τ_R the process X is a solution of the equation

$$X_{t\wedge\tau_R}=x+\sum_{j=1}^{\infty}\int_0^{t\wedge\tau_R}\sigma_j(s,\omega,X_s)\,dW_s^j+\int_0^{t\wedge\tau_R}b(s,\omega,X_s)\,ds,$$

where the coefficients σ_j and b are adapted, locally bounded and $(t,x)\to\sigma_j(t,\omega,x)$ are Lipschitz continuous and satisfy

$$\gamma_t\geq\sigma\sigma^*(t\wedge\tau_R,\omega,X_{t\wedge\tau_R})\geq\lambda_t.$$

We obtain lower bounds of the form:

$$\exp\left[-Q_n^1\left(1+\int_0^{T+r}F_x^1(t)\,dt\right)\right]\leq P(\tau_R>T) \tag{1}$$

where Q_n^1 is a constant and F_x^1 is a function that depends on x_t and R_t.

We apply the results to obtain lower bounds for option prices for stochastic volatility models.

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