Partial Differential Equations
Equations aux dérivées partielles
(Org: **Monica Clapp** (UNAM), **Nassif Ghoussoub** (UBC) and/et **Pablo Padilla** (UNAM))

WALID ABOU-SALEM, University of British Columbia
Mean-field dynamics of rotating bosons in confining traps

I discuss the rigorous derivation of the time-dependent magnetic Hartree equation that describes the mean-field dynamics of rotating bosons in confining traps.

NILS ACKERMANN, UNAM, Circuito Exterior, CU, México, D.F., C.P. 04510
Alternating sign multibump solutions of nonlinear elliptic PDEs in expanding tubular domains

Let Γ denote the image of a smooth embedding of the circle S^1 in $\mathbb{R}^N$, $N \geq 2$. Denote by Ω_R the open normal tubular neighborhood of $R\Gamma$ of radius 1. Consider the superlinear problem $-\Delta u = f(u)$ on the expanding domains Ω_R (i.e., as $R \to \infty$) with homogeneous Dirichlet boundary conditions. We prove the existence of multibump solutions with bumps lined up along $R\Gamma$ and with alternating signs. Here we allow nonodd functions f.

ANTONIO CAPELLA-KORT, Universidad Nacional Autonoma de Mexico
On a perturbation of the linearized two dimensional two-well problem

The two dimensional two-well problem arises in the study of the zero energy states of a solid-solid phase transition in materials that exhibit the so-called shape memory effect.

This problem can be formulated as follows: find $u\colon \Omega \to \mathbb{R}^3$ such that

$$\nabla u \in K = \mathrm{SO}(2)U_a \cup \mathrm{SO}(2)U_b, \quad \text{a.e. in } \Omega, \tag{1}$$

where U_a and U_b are two traceless symmetric matrices, and $\mathrm{SO}(2)$ represents the set of proper rotations. The most simple nontrivial solution to (1) is given by the so-called simple laminates, that is, the function u depends only on one cartesian coordinate. In fact, it was show by Dolzmann and Muller that if the perimeter of the transitions is finite, u has to be simple laminate.

For a suitable energy and in the proper regime, we show that the nonzero energy states for the linearized version of (1), are also closed to a simple laminate.

DAVID COSTA, University of Nevada Las Vegas, 4505 Maryland Pkwy, Box 454020, Las Vegas, NV 89154-4020, USA
Sharp Constants and Minimizers for a Class of Inequalities

We consider a class of Caffarelli–Kohn–Nirenberg inequalities without restricting the pertinent parameters and determine the values of the optimal constants and the functions that achieve them, i.e., minimizers of a suitable functional. By studying a corresponding Euler–Lagrange equation, we also find infinitely many sign-changing solutions at higher energy levels in addition to the ground-state solutions.

CRAIG COWAN, University of British Columbia, Vancouver, BC
A fourth order equation modelling a simple MEMS device

We examine the equation

$$\Delta^2 u = \frac{\lambda}{(1-u)^2} \quad \text{in B,}$$

with zero Dirichlet boundary conditions where B is the unit ball in $\mathbb{R}^N$ and where $\lambda > 0$ is a parameter. We show that the *extremal solution*, u^*, is smooth provided $N \leq 8$ and singular for $N \geq 9$.

This equation is of practical interest since it is the steady state of an equation modeling a simple Micro-Electro-Mechanical System (MEMS) device.

MOHAMMAD EL SMAILY, University of British Columbia
Speed of propagation for reaction-diffusions with large drift

[no abstract supplied]

CARLOS GARCÍA-AZPEITIA, Depto. Matemáticas y Mecánica, IIMAS–UNAM, Apado. Postal 20-276 01000, México DF
Applications of the ortogonal degree to the problem of bifurcation in Hamiltonian dynamical systems

We investigate the bifurcation of periodic solutions from relative equilibria, examples being the n body problem or the n vortex problem. We use the approach of orthogonal degree theory, which lets us probe the existence of global symmetric branches of periodic solutions. We particularly report a general result of bifurcation on the equation of a satellite influenced by a relative equilibria of primaries. We will discuss further the case in which the primaries form a $1+n$-gon, like the Maxwell model for the Saturn rings. We also discuss the case of Halo orbits in the restricted tree body problem.

MAURICIO LABADIE, University of Paris VI
A reaction-diffusion model for calcium in neurons

The concentration of calcium ions inside dendritic spines (microstructures of the neuron) plays a crucial role in the synaptic plasticity, and in consequence in cognitive processes like learning and memory. We construct a reaction-diffusion system that models the dynamics of calcium ions in the spine, taking into account the chemical interactions between the calcium ions and three different types of proteins. We prove that this system is a well-posed problem, i.e., we have a priori estimates, global existence, global uniqueness, positivity of solutions and continuity with respect of the initial data.

This result will appear in the article of Kamel Hamdache and Mauricio Labadie, *On a reaction-diffusion model for calcium dynamics in dendritic spines*, Nonlinear Analysis: Real World Applications **10**(2009), 2478–2492 (August issue). This article has been published online on May 2008 (doi: 10.1016/j.nonrwa.2008.05.005).

GABRIEL LÓPEZ-GARZA, Universidad Autónoma Metropolitana, Mexico DF
Existence and Multiplicity for a Resonance Problem for the p-Laplacian

The existence of at least two solutions for a resonance problem involving the p-Laplacian is shown for the case of bounded domains in $\mathbb{R}^N$. This work constitutes an extension of a previous result of Landesman, Robinson and Rumbos for the case $p = 2$ (*Nonlinear Analysis TMA* **24**(1995), 1049–1059).

ALIP MOHAMMED, York University
Poisson equation with the Robin boundary condition

The inhomogeneous Robin/third boundary condition with general coefficient for the Poisson equation on the unit disc is studied in terms of holomorphic functions using Fourier analysis. It is shown that against the usual expectations this problem

cannot have a unique solution unless the coefficient of the first order term in the boundary condition is a constant. For the case of general coefficient, it is actually a problem with essential singularity in the domain, but still well-posed under proper assumptions and the unique solution is given explicitly.

AMIR MORADIFAM, University of British Columbia

Bessel pairs and optimal Hardy and Hardy–Rellich inequalities

We give necessary and sufficient conditions on a pair of positive radial functions V and W on a ball B of radius R in R^n, $n \geq 1$, so that the following inequalities hold for all $u \in C_0^\infty(B)$:

$$\int_B V(x)|\nabla u|^2\,dx \geq \int_B W(x)u^2\,dx,$$

and

$$\int_B V(x)|\Delta u|^2\,dx \geq \int_B W(x)|\nabla u|^2\,dx + (n-1)\int_B \Big(\frac{V(x)}{|x|^2} - \frac{V_r(|x|)}{|x|}\Big)|\nabla u|^2\,dx.$$

This characterization makes a very useful connection between Hardy-type inequalities and the oscillatory behaviour of certain ordinary differential equations, and helps in the identification of a large number of such couples (V, W)—that we call Bessel pairs—as well as the best constants in the corresponding inequalities. This allows us to improve, extend, and unify many results—old and new—about Hardy and Hardy–Rellich type inequalities, such as those obtained by Caffarelli–Kohn–Nirenberg, Brezis–Vázquez, Wang–Willem, Adimurthi–Chaudhuri–Ramaswamy, Filippas–Tertikas, Adimurthi–Grossi–Santra, Tertikas–Zographopoulos, and Blanchet–Bonforte–Dolbeault–Grillo–Vasquez. As an application we give a mathematical proof for the singularity of the extremal solution of the bilaplacian with exponential nonlinearity in dimensions $N \geq 13$.

PANAYOTIS PANAYOTAROS, UNAM

Continuation of breathers in a finite discrete NLS lattice

We present some preliminary results on the continuation and bifurcations of breathers in the discrete NLS in a finite one-dimensional lattice. We show numerical evidence for fold and pitchfork bifurcations as the intesite coupling increases and also discuss breathers that can be continued to normal modes of the weakly linear system.

CARLOS VÉLEZ, Universidad Nacional de Colombia–Sede Medellín

Existence of sign-changing solutions for resonant and non-resonant semilinear elliptic problems

In this talk we briefly present recent results regarding the existence of sign-changing solutions of the semilinear elliptic problem

$$\begin{cases} \Delta u + f(u) = 0 \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain and $f\colon \mathbb{R} \to \mathbb{R}$ is a nonlinear C^1-function which is also *asymptotically linear*, i.e.,

$$\lim_{|t|\to\infty} f'(t) \in \mathbb{R}. \tag{2}$$

Our results rely on two main tools: first, a precise *a priori* estimate for signed-solutions of (1). Second, an abstract Lyapunov–Schmidt reduction method, which allows us to get solutions with relatively large augmented Morse indexes. These tools are applicable to both resonant and non-resonant cases.

The most of these developments are a joint work with Alfonso Castro and Jorge Cossio.

RAMÓN ZÁRATE, UBC

Inverse problems via variational methods.

We present a general variational method for recovering non-linearities from prescribed solutions for certain types of PDEs which are not necessarily of Euler–Lagrange type, including parabolic equations. The approach can be used for optimal control problems.