JOSE SEADE, UNAM-Cuernavaca Foliations with Bott-Morse Singularities

This talk will be about joint work with Bruno Scardua (Brazil) on codimension one foliations on oriented smooth manifolds having a singular locus where the foliation is locally defined by a Bott-Morse function. Examples of such foliations arise, for instance, by considering the fibers of a Morse function f on a smooth manifold M and their liftings to manifolds which are fiber bundles over M. Another class of examples of such foliations arises by looking at cohomogeneity 1 actions of compact Lie groups on smooth manifolds having exceptional orbits. There are many other ways how foliations with Bott-Morse singularities arise.

We show how the classical theory for non-singular foliations, such as Reeb's stability theorems, extends to singular foliations of this type. In the particular case when all the singularities are transversally centers, our stability theorem yields to a topological characterization of all such foliations. We thus get a theorem that unifies the classical theorem of Reeb, that a manifold with a Morse foliation with only center-type singularities must be the sphere, and the well-known theorem that a cohomogeneity 1 action of a compact Lie group having exceptional orbits has exactly two such orbits and the orbit space is the interval.

We also prove a stability theorem for Bott-Morse foliations with saddle-singularities and various topological consequences of it.