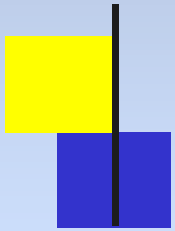


Making School Mathematics Functional

a stool needs three legs

Hugh Burkhardt

Shell Centre, University of Nottingham

Canadian Mathematics Education Forum
Vancouver, May 2009

Structure

- Mathematics that is functional
- Performance goals in Mathematics
- The three legs of systemic change
- Task design: issues, strategies, tactics
- Teaching and teaching materials
- Supporting professional development

The Shell Centre Team

- Malcolm Swan, Daniel Pead, Rita Crust, Alan Bell, HB, with many associates
- Tool design engineers doing engineering research in education, ie design and development of:
  - teaching materials and processes
  - assessment tasks
  - professional development materials and processes
  - tools and strategies for system change

  with some associated ‘insight research’
- Based in the University of Nottingham School of Education
- Works with many others, notably Berkeley, Michigan State, and school systems in UK and US
- Contact: **Hugh.Burkhardt@nottingham.ac.uk**
  www.mathshell.com

Functional Mathematics

Non-specialist adults,
if they are taught how,
benefit from using mathematics in their everyday lives to better understand the world they live in, and to make better decisions.

"The sophisticated use of, often elementary, mathematics"
also called
mathematical literacy (ML), quantitative literacy, numeracy
…

Post-age-11 mathematics is non-functional for most people

“PONZI” PYRAMID SCHEMES

Max has just received this email

From: A. Crook

To: B. Careful

Do you want to get rich quick?
Just follow the instructions carefully below
and you may never need to work again:

1. Below there are 8 names and addresses.
   Send $5 to the name at the top of this list.
2. Delete that name and add your own name and
   address at the bottom of the list.
3. Send this email to 5 new friends.

"PONZI" PYRAMID SCHEMES

- If that process goes as planned,
  how much money would be sent to Max?
- What could possibly go wrong?
- Why do they make Ponzi schemes like this illegal?

builds understanding of standard scam – sees the power of exponential growth, and why it can't go on for ever

Making a case

The spreadsheet contains 2 sets of reaction times, 100 each for Joe and Maria.

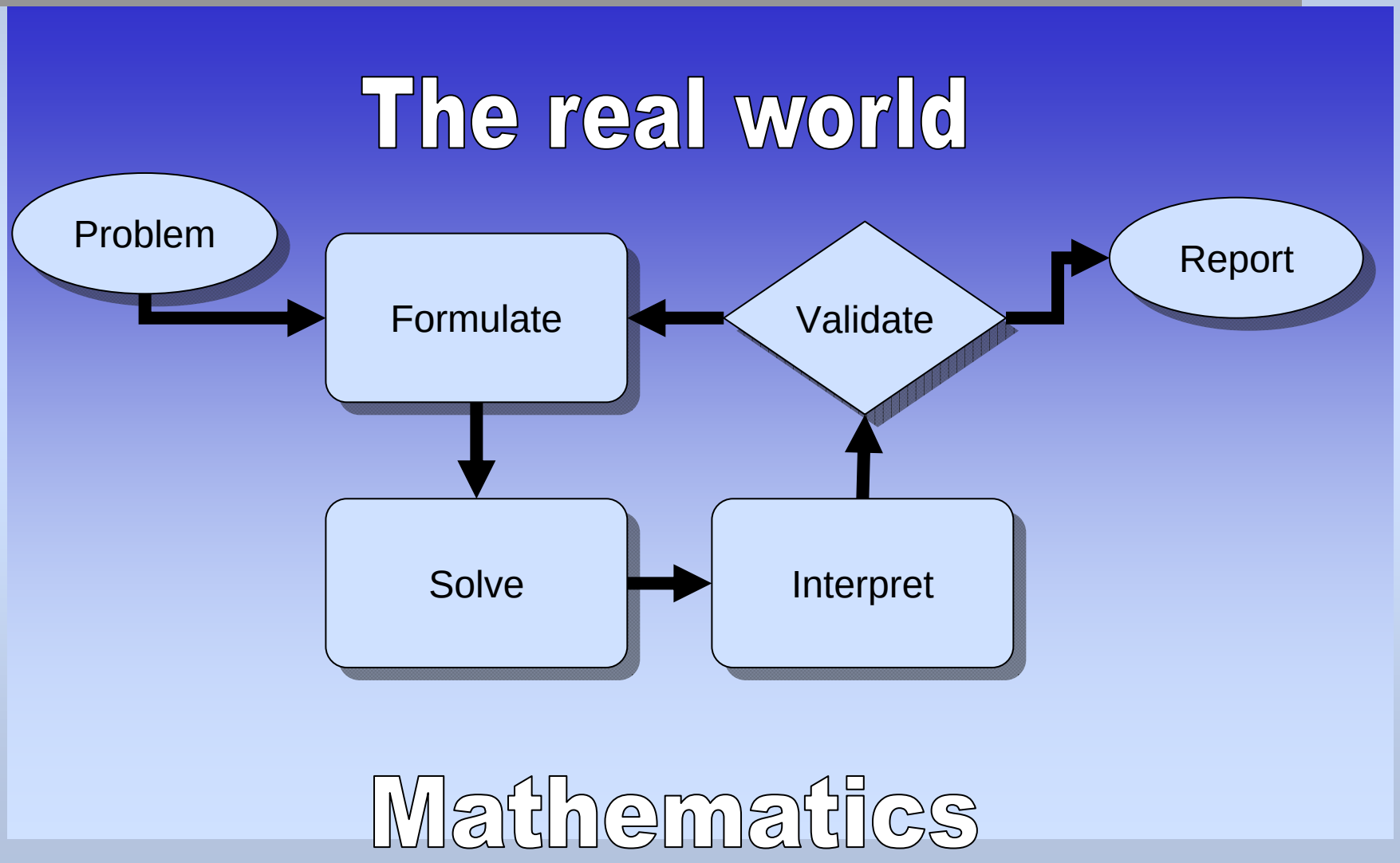
- Using this data, construct two arguments:
  - A: that Joe is quicker than Maria

  and

  - B: that Maria is quicker than Joe

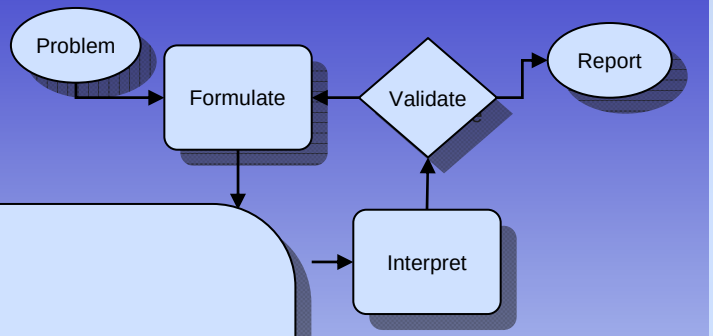
builds understanding, and intelligent scepticism, of how political and marketing data is used – uses different summative measures on the same data

The modelling process



Dysfunctional math curricula

The real world



Solve

Mathematics

What does ML involve?

- “The sophisticated use of, often elementary, mathematics”
- All key aspects of ‘doing mathematics’
  - Beliefs
  - Strategies
  - Techniques
  - Metacognition
  - Control
- “The Few Year Gap”
  between imitation and autonomy

cf Specialist Mathematics

SM provides the mathematical toolkit for further study in socially important fields: engineering, physics, economics, ….

into which an important minority will go.

SM shows more of the intellectual excitement of mathematics (cf music)

Here I will focus on functional mathematics because if its:

- Social importance for all
- Motivation for most

Specialist mathematics, done properly, needs all the same things.

Modelling

- Joe buys a six-pack of coke for $3 to share among his friends. How much should he charge for each bottle?
- If it takes 40 minutes to bake 5 potatoes in the oven, how long will it take to bake one potato?
- If King Henry 8th had 6 wives, how many wives had King Henry 4th?

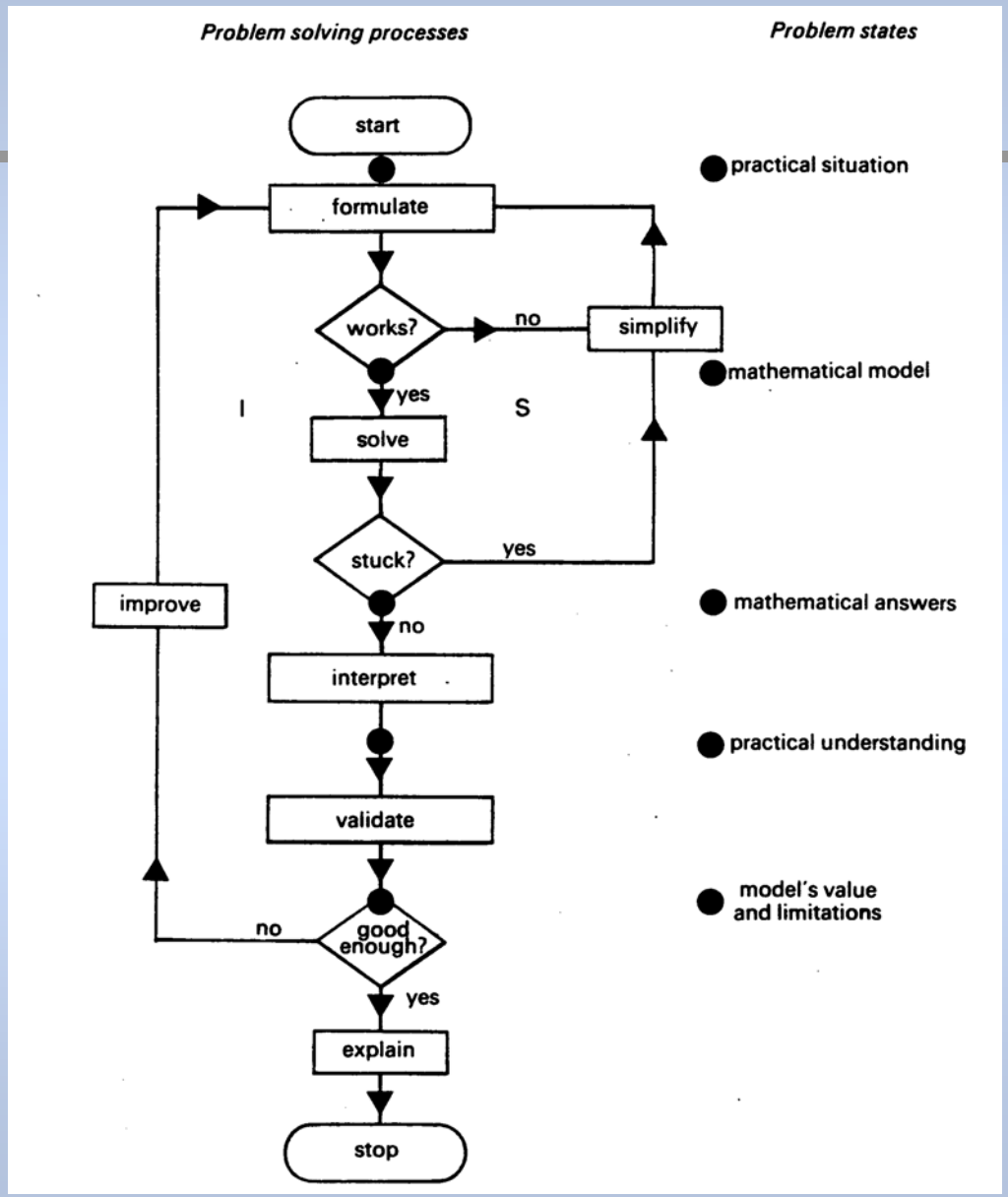
Teaching modelling: some history

- 1960- individual experimental courses

Scale of implementation, mainly UK and US

- 1970-90 some UG courses (ICTMAfia)
- 1990- –ve progress/cosmetic realism
- Now: in some Germany (regions) a coherent move to establish modelling

  England: adopts “functional maths” – meaning unclear


Problem solving processes
Problem states
start
formulate
works?
no
simplify
yes
I
S
solve
stuck?
yes
improve
no
interpret
validate
no
good enough?
yes
explain
stop
practical situation
mathematical model
mathematical answers
practical understanding
model's value and limitations

Present situation?

- If you drop into 100 randomly chosen mathematics classrooms, will you see modelling? Unlikely

Why? Unsolved problem but …

- Broader teaching skills than imitative curriculum
- Mathematics remains inward-looking
- Deep change needs pressure and support

- Don't give up

Research >large scale practice 25 years

- Penicillin, vacuum cleaner, gene therapy
- Systemic change makes it harder

The three legs of the stool

- Assessment
- Professional development
- Teaching materials

How do we get them balanced?

What kinds of
tools and processes
do we need to make this happen?

Pressure + Support

- System and culture dependent
- Pressure: good or bad
  - Anglos: high-stakes tests + National Curriculum + inspections
  - *What is it in your province/state/country?*
  - PISA?
- Support
  - Teaching materials
  - Professional development
  - *What is it in your country?*

To work, these must be well-engineered+aligned

Professional development pathway

0. Managing the class
1. Delivering the textbook
2. Adding good activities (eg NCTM)
   Many teachers “plateau” here
   For some teachers, this routine expertise then develops into adaptive expertise
   (Hatano, Schoenfeld)
3. Building on where each student is
   Catalyzing and supporting that shift is the core challenge of PD, involves changing:
   1. Knowledge – of math and pedagogy
   2. Orientation – the “classroom contract”
   3. Goals – dimensions of performance

Issues in task design

- The roles of assessment
- Performance goals in Mathematics
- Task design principles
- Task design: issues, strategies, tactics
- Building tests within constraints

Roles of high-stakes assessment

Role A: Measures levels of performance

Role B: exemplifies performance objectives

Role C: determines classroom activity

Standard errors:
- only consider A
- rely on correlation

(Paleo-)Psychometrics ignores *what* is assessed

What design responsibilities do A+B+C imply?

The importance of good tasks

show performance goals in a compact way

Types of mathematical task

- reproduce a learned procedure
  such ‘exercises’ now dominate
- critique and improve
- plan
- design
- evaluate and recommend
- investigate
- …..

Plan a trip: fault finding and fixing

Alison and two friends has planned a cycling trip around Derbyshire on Saturday.

Here is their plan for the day.

Read through the plan and the information sheets (next page).

If you find a mistake, or realise something has been forgotten, write it down and say how they should change the plan.

Meet at Loughborough station at 7.23 am. Buy tickets and then catch the train to Derby. This arrives at 7.51 am.

At Derby, catch the 8.20 am train to Cromford. This arrives at 8.41 am.

Here are the instructions for getting to the Cycle Hire centre:

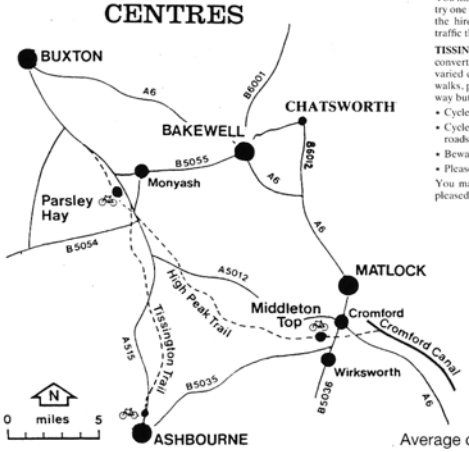
“Turn left as you come out of Cromford station, walk along by the river and down Mill road. Cross over the A6. Walk up Cromford hill for about 1/2 mile and you will see..

Authentic information sheets

Cycle hire information

CYCLE HIRE CENTRE	LOCATION	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEPT	OCT
MIDDLETON TOP End of season Cycle Sales November 5th Tel: (062 982) 3204	Middleton Top Picnic Site & Information Centre, on the High Peak Trail next to the Old Engine House. Signposted off the B5036 Cromford to Wirksworth road. OS MAP 119 Grid Reference SK276552	Open for Bookings only	Open for Bookings only	Open 28 to 31 & Bookings	Open 1 to 6, 12 & 13, 19 & 20, 26 & 27 & Bookings	Open 3 to 5, 10 & 11, 17 & 18, 24 to 31 & Bookings	Open every day	Open every day	Open every day	Open 1 to 7, 13 & 14, 20 & 21, 27 & 28 & Bookings	Open weekends & Bookings
HAYFIELD	Hayfield Station Picnic Site on the Sett Valley Trail, just off the A624 Chapel en le Frith-Glossop road.	Open for Bookings	Open for Bookings	Open 28 to 31	Open 1 to 6, 12 & 13	Open 3 to 5, 10 & 11	Open 1, 7 & 8, 14 & 15	Open every day	Open every day	Open weekends & Bookings	Open for Bookings

CYCLE HIRE CENTRES



WHERE TO?

You may follow your nose along the country roads and bridleways, or try one of the circular routes that have been waymarked from some of the hire centres. OR you may prefer the complete lack of motor traffic that can be enjoyed if you cycle exclusively on the trails.

TISSINGTON and HIGH PEAK TRAILS – The trails have been converted from former railways to form fine scenic routes through varied countryside, where visitors can enjoy a picnic, long and short walks, pony trekking, as well as cycling. Explore the trails in your own way but please remember:

- Cycles must give way to pedestrians on the trails
- Cycles are not permitted on footpaths: you can go anywhere on roads and bridleways.
- Beware of traffic especially on main roads.
- Please follow the Country Code.

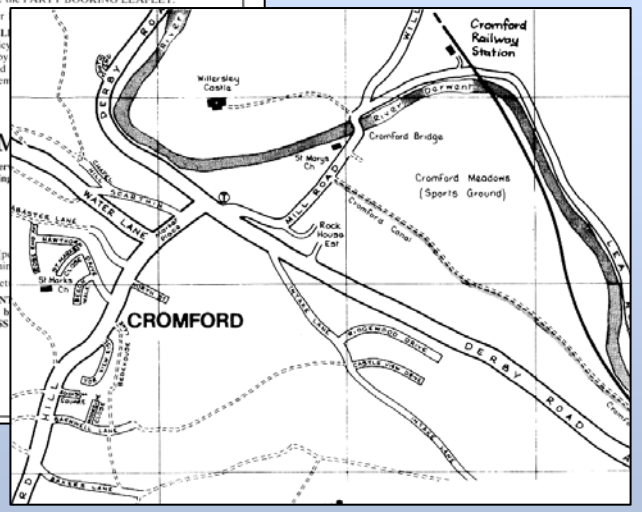
You may meet Countryside staff working on the trails who will be pleased to help you with any further information.

Average cycling speed is about 8 mph.

Based upon a map produced by the Derbyshire Dales District Council (Crown Copyright reserved) and on information from Peak District National Park.

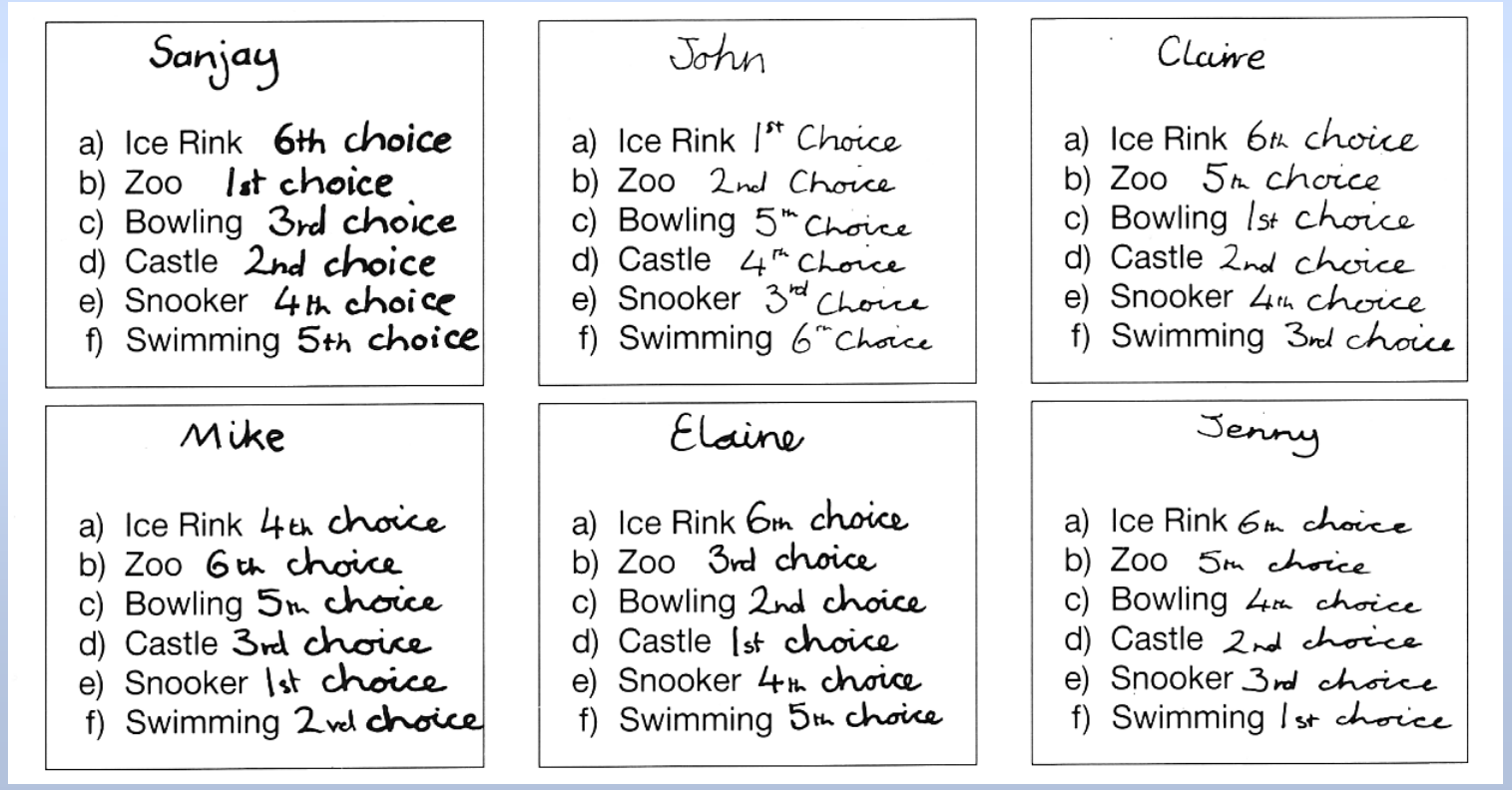
Saturdays

									A			
Sinfin Central ..... d	....	....	....	....	....	....	....	....	....	....	....	....
Sinfin North §..... d	..	..	..	..	..	..	..	..	..	..	..	..
Peartree..... d	....	....	....	....	....	....	....	....	....	....	....	....
Derby..... a	..	..	..	..	..	..	..	..	..	..	..	..
London St. Pancras ..... ⇔ 53 d	....	....	0745	0910	....	1135b	1235b	1515b	1630b	....	2105	....
Nottingham ..... 80 d	..	0731	0844	1033	1153	1354	1453	1650	1800	..	2138	..
Derby ..... d	0630	0820	0947	1115	1235	1422	1523	1724	1832	2032	2312	....
Duffield ..... d	0637	0827	0954	1122	1242	1429	1530	1731	1839	2039	2319	..
Belper ..... d	0641	0831	0958	1126	1246	1433	1534	1735	1843	2043	2323	....
Ambergate..... d	0647	0837	1004	1132	1252	1439	1540	1741	1849	2049	2329	..
Whatstandwell ..... d	0651	0841	1008	1136	1256	1443	1544	1745	1853	2053	2333	....
Cromford ..... d	0656	0846	1013	1141	1301	1448	1549	1750	1858	2058	2338	..
Matlock Bath ..... d	0658	0848	1015	1143	1303	1450	1551	1752	1900	2100	2340	....
Matlock ..... a	0701	0851	1018	1146	1309	1453	1554	1755	1903	2103	2343	..



Plan a trip: voting

Six people are planning a day out.
Six different places have been suggested:
Ice rink; Bowling alley; Swimming pool; Zoo; Castle; Snooker hall
They take a vote. Which would be the best place for the trip and why?



Sudden Infant Deaths = Murder?

- In the population as a whole, about 1 baby in 8,000 dies in an unexplained "cot death". The cause or causes are at present unknown.
- Three successive babies in one family have died.
- The mother is on trial. An expert witness says:

  "One cot death is a family tragedy; two is suspicious; three is murder. The odds on three deaths in one family are 64 million to 1"

Discuss the reasoning behind the expert witness' statement, noting any errors, and write an improved version to present to the jury.

Task realism

I have found it useful to distinguish

A Action problems – for now

B Believable problems – for the future

C Curious problems – for delight

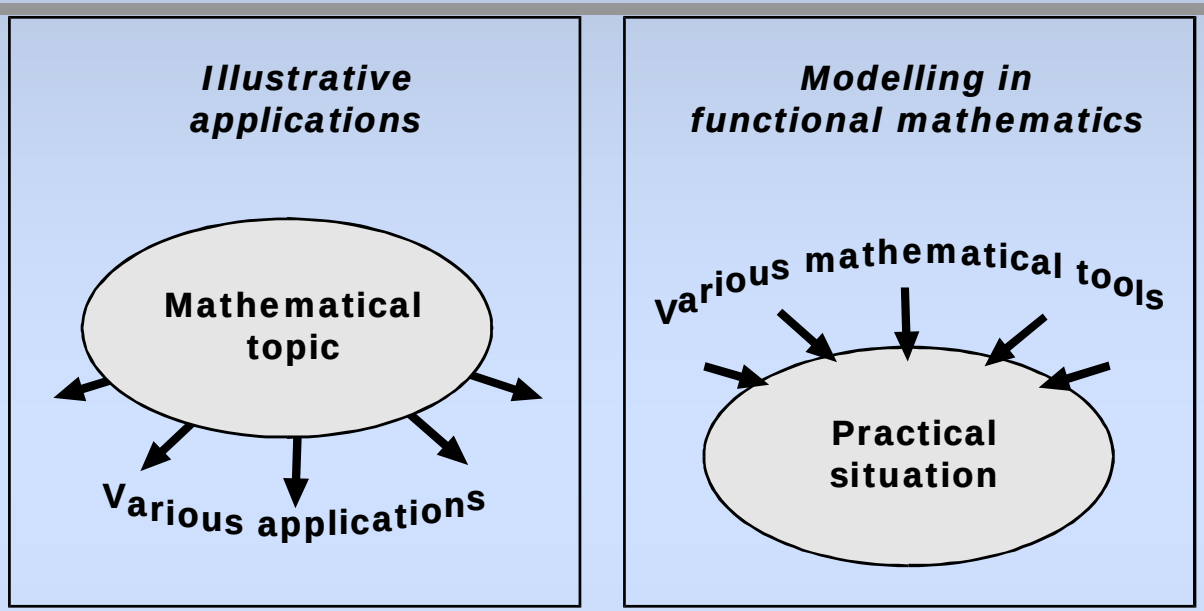
D Dubious problems (look in any math book)

E Educational problems – D but OK

Dimensions of performance

- Content: math topics, concepts, skills
- Phases of problem solving/modeling
- Non-routine-ness
- Open-ness: closed, open middle, end
- Goal type: applied power, pure math
- Reasoning length
- Task type

An important distinction



- Illustrative applications show standard models
- Active modelling of situations you know well, but have not previously analysed, is essential for ML

Task difficulty

Depends on a combination of

- Complexity
- Unfamiliarity
- Technical demand
- Student autonomy

Cannot be reliably predicted, hence trials

"Few year gap" v imitative exercises

Making Soft Toys

- Sue and Terry are making dogs and teddy bears.
- They have time to make 18 toys, and £60 to spend on materials
  Materials for a dog cost £3, materials for a teddy bear cost £4.
  They sell each dog for £8 and each teddy bear for £10.
- How many of each should they make to maximise profit?

Long jump

- Three girls compete to be selected for the regional long jump competition.
- Each has six jumps; the results are shown in the table.
- Which girl should be selected? Explain your reasoning.

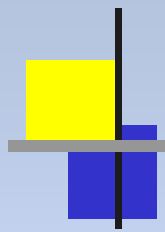
from TIMSS video study

Elsa	Ilse	Olga
3.25	3.55	3.67
3.84	3.99	3.78
4.10	3.61	3.92
2.95	3.97	3.62
3.66	3.69	3.85
3.86	3.59	3.73

Long jump

- They calculated the average jump for each girl !! >> Olga
- The teacher moved on
- There was no discussion of other/appropriate measures – a worthwhile task.

Elsa	Ilse	Olga
3.25	3.55	3.67
3.84	3.99	3.78
4.10	3.61	3.92
2.95	3.97	3.62
3.66	3.69	3.85
3.86	3.59	3.73



Mathematics uses Computers

everywhere……. except in school math

- Computers are valuable tools for:
  - organising data, and thinking – spreadsheets
  - finding information – via the web…
  - simulating real world problems
  - doing + checking messy procedures…….

.. but school implementation is challenging

  - timescale mismatch
  - equity concerns
  - teacher skills
- Modularising may help

Who needs it?

Plan

- Potential secretaries asked to critique and complete the spreadsheet for planning a conference budget

Graduates who "know Excel" don't create formulas in col E

A	B	C	D	E
	College charges	Delegates	@ £ each	£
Monday	Buffet Supper	30	17.00	0
	Single En-suite Accommodation	30	40.00	0
Tuesday	Breakfast	30	8.00	0
	Morning Coffee	30	1.90	0
	Luncheon	30	15.00	0
	Afternoon tea	30	1.90	0
	Dinner served	30	50.00	0
	Single En-suite Accommodation	30	40.00	0
	Plenary Room	30	15.77	0
	Breakout rooms	2	85.10	0
Wednesday	Breakfast	30		0
	Morning Coffee	30		0
	Luncheon	30		0
	Afternoon tea	30		0
	No Dinner	30		0
	Single En-suite Accommodation	30		0
	Plenary Room	30		0
	Breakout rooms	2		0
Thursday	Breakfast	30	8.00	0
	Morning Coffee	30	1.90	0
	Luncheon	30	15.00	0
	Afternoon tea	30	1.90	0
	Dinner	30	17.00	0
	Single En-suite Accommodation	30	40.00	0
	Plenary Room	30	15.77	0
	Breakout rooms	2	85.10	0
Friday	Breakfast	30	8.00	0
			Total charges	0
			VAT	0
			Total	0

Tree Rings

Tree rings

Page 1 | Page 2 | Page 3 | Page 4

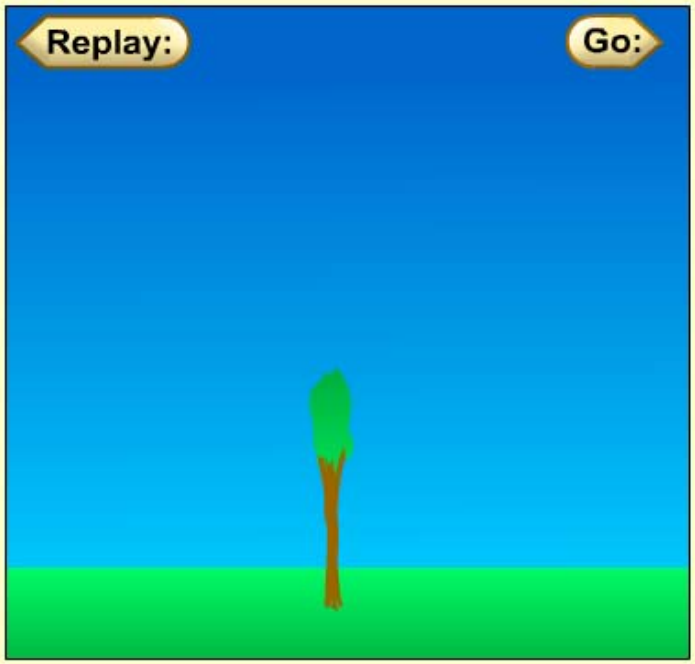
Trees grow by making new wood just under their bark.

Each year, a new layer of wood is added.

When a tree is cut down, you can see these layers as a pattern of rings.

You can tell the age of the tree in years by counting the rings.

Press "Go" and watch the movie.



1. The tree in the movie was cut down in the year 2000.

   In what year was it planted? ?

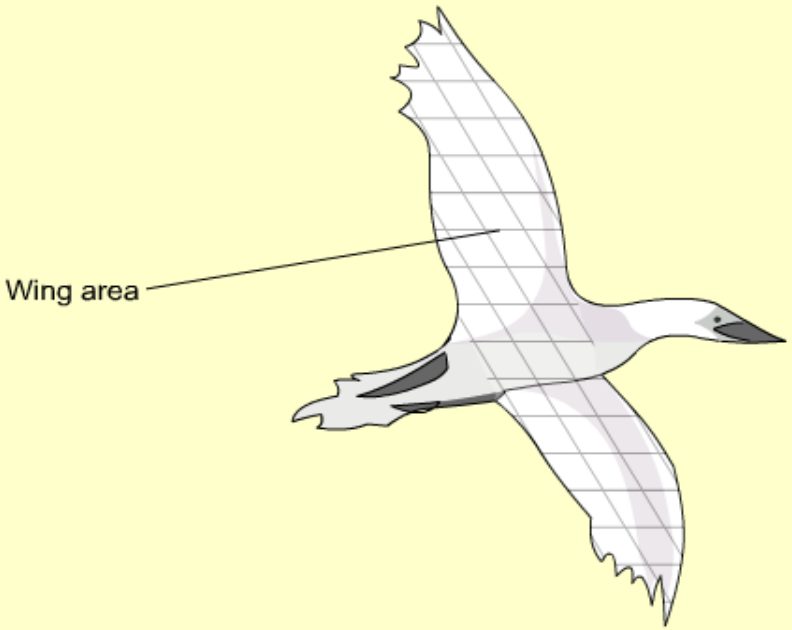
   In what year did it grow the most? ?

Fly Fast

Fly fast

Sandra and Ian have collected data on different types of bird.

They know how fast they fly, how much they weigh and the area of their wings.



They want to know if they can predict how fast a bird will fly from its size and shape.

Now go on to page 2

Teaching Math Literacy

For Mathematical Literacy units so far, it seems:

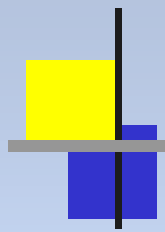
- All students succeed and enjoy the work
- ML narrows the range of performance
- Many, but not all, teachers can handle this work with just the materials – more with live PD training
- 1 or 2 new three-week units per year is digestible

More research needed, across more exemplar units to warrant such general statements

“Bowland Maths”

- ~20 “case studies, including:
  - Reducing road accidents
  - How risky is life?
  - “You reckon?”
  - Alien invaders
- Professional development
  - 5 module package, activity based
- Assessment

The importance of design + engineering



What do good designers do?

They know how to

- use research results and design skills to
  - improve ‘best practice’
  - tackle new challenges effectively
- pass on their knowledge to
  - other practitioners
  - novices

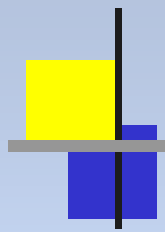
through their materials.

Educational design principles

Heuristic, phenomenological theory:

- Some based on ‘insight research’, eg
  - active learning
  - constructive
  - build multiple connections
- Others design-based, eg
  - role shifting
  - cognitive conflict
  - student ‘ownership’

Design theory is not often discussed
in enough detail to be useful



Design beyond just principles

Design brilliance is more than these:

- ‘Surprises’ that are clearly ‘right’
- Handling complexity simply
- Controlled innovation
- Balance in all aspects

We know it when we see it – iPod,...

What development skills are needed?

- The team needs:
  - Systematic methods of observation
  - Interview skills
  - Protocols related to the design goals
  - Methods for analysing observation reports, student work, interviews
- Design skill in using this rich feedback systematically to improve the materials.

ie as products are developed in other fields

Design > Engineering research

Design Research has emerged as an accepted part of educational research, with a strong input from Cognitive Science. Key features include:

- insight focus > products and papers
- realistic classroom situations
- exploring teaching and learning
- theory building

but with

- atypical teachers
- exceptional support
- no claim to wider usability >no direct impact

Engineering research: these products are drafts

Design research > Engineering

For more, see e.g.

Educational Design Research

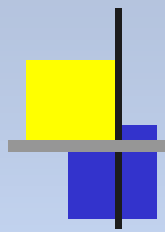
eds Jan van den Akker, Koeno Gravemeier, Susan McKenney, Nienke Nieveen

Routledge 2006

"pragmatic, grounded, interactive, iterative and flexible, integrative, and contextual"

Who does it?

Why isn't it the mode of development?

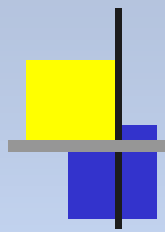


“Authors” and publishers

See no need or justification, because

- systematic evaluation is non-existent
- good engineering
  - costs much more
    ~ $20,000 per teaching hour
    still neglible cf system running cost
  - takes time
- powerful tools require more skill

Education ~ “alternative medicine”



Academics?

But the value system favours:

- new ideas *over* reliable research
- new results *over* replication and extension
- trustworthiness *over* generalizability
- small studies *over* major programs
- personal research *over* team research
- first author *over* team member
- disputation *over* consensus building

papers *over* products and processes

International Society for Design and Development in Education

- www.isdde.org

  ISDDE Conference 2009
  Cairns, Queensland, Australia
  September 27th-30th 2009
  Contact:
  Kaye Stacey, Conference Chair
  k.stacey@edfac.unimelb.edu.au
  www.isdde.org

  see also

- *Educational Designer*, an e-journal

Teaching materials

- What does your ‘scheme’ cover?
- Who is it designed for? *(realistically!)*
- Moving beyond the published ‘scheme’
  - Selecting replacement units
  - Learning through misconceptions
  - Maintaining some coherence
    – but not too much

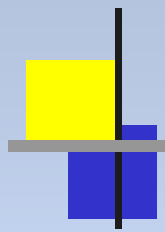
Issues for curriculum design

- Is this outward-looking mathematics?
  - few students will become mathematicians
  - math can give them power in their lives
  - does this curriculum do that? for all? (cf ELA)
    or is it “just math” (RPF)

  symptoms: all topic focus, no modelling, tasks

- What ‘dimensions of engagement’?
  - many students lack interest in math itself
  - is “make the math interesting” all this does?
  - Does it build ‘mathematical power’

  symptoms: variety of activities, of tasks (cf ELA)



… and a few more design issues

- Does this develop student autonomy?
  - reliable imitation is not enough to do math
  - what ‘transfer distances’ do the tasks cover?
  - how long are the chains of reasoning?
  - …involving, which problem solving phases?

  symptoms: no linked phases, similar tasks together

- Does this give teachers enough support?
  - Student-centered teaching is difficult
  - it is easy to overload the teacher
  - what design tactics are used to avoid this?

  symptoms: teacher in hot seat, centre-stage; no support tactics; too much innovation at once; ……

Professional development pathway

0. Managing the class
1. Delivering the textbook
2. Adding good activities (NCTM)
   Many teachers “plateau” here
   For some teachers, this routine expertise then develops into adaptive expertise
   (Hatano, Schoenfeld)
3. Building on where each student is
   Catalyzing and supporting that shift is the core challenge of PD, involves changing:
   1. Knowledge – of math and pedagogy
   2. Orientation – the “classroom contract”
   3. Goals – dimensions of performance

Professional development

- Needs to be materials based, because:
  - Ratio skilled trainers/needy teachers ~ 0.001
  - Needed for TTT ‘cascade’ to work
- Design principles:
  - Activity based
  - General principles from specific exemplars (constructive teacher learning)
  - Ongoing, few-year timescale
- Key foci
  - Handling non-routine problems in the classroom
  - Handling discussion non-directively
  - Questioning
  - Changing the “classroom contract”: roles, expectations

Progress so far

- ML can be taught by normal teachers, e.g.
  - Numeracy through Problem Solving (Shell Centre,1988)
  - Realistic Mathematics Education (FI, 2000)
  - Bowland Maths (UK groups 2008)
- Research-based design pays off:
  - The above examples
  - US evaluation evidence,…..
- Major challenges:
  - Getting the three legs balanced
  - Dynamics of system change