Working Group Why is Mathematics Relevant in Our Society?

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The question of relevance is an important one for mathematics educators. As simple as it is to pose, it is very complex to address. The Forum organizers suggested that the objectives of our working group could be to:

- Identify some good examples which illustrate both the relevance and the feasibility in a classroom setting.
- Address some of the less conventional connections with respect to relevance, namely in other human creative endeavours like visual arts, dance and music.
- Document resources that address this question.
- Ask, in what way to do these examples fit in the school mathematics program? How would things have to change so they did?

There are a number of perspectives that can be used from which to respond to the question of relevance, and as working group leaders we brought very particular perspectives. For example, in preparing for the working group Peter and Elaine shared their views on the relevance of mathematics in society with each other. Whereas Elaine wanted to emphasize the relevance of mathematics that arises for citizenship, Peter felt the need to approach relevance from an aesthetic perspective. On the other hand, both Peter and Elaine felt it was important to bring examples of what they meant by this to the working group. Hence the working group was structured in the following way: Day1– Elaine raised questions of mathematics for citizenship (this included the question of numeracy or quantitative literacy); Day 2– Peter raised the question of beauty and good problems for mathematics instruction; Day 3– members of the group discussed their perspectives on the relevance of mathematics to society.

Day 1

A mathematician should never forget that mathematics is too important to frame its instruction to suit more or less the needs of future mathematicians. (Freudenthal)

On the first day of the working group, we addressed relevance from two perspectives: the first was the perspective that mathematics is needed for people to fully participate in society and the second was from a student's perspective in terms of relevance for meaning making.

We began the working group by offering a prompt intended to express and share some of our fundamental attitudes about the relevance of mathematics. Rather than simply introduce ourselves, participants were asked to complete the analogy: If mathematics were part of the ecosystem it would be the _____. There were a variety of responses including: the air– it is everywhere, the water cycle, the earth, the sun, and it is about relationships, how the different components fit together.

The participants' responses give us some indication of their views on the relevance of mathematics: mathematics is well-connected and highly linked; it permeates our society; it is integral to the functioning of society through the role it plays in science, economics and even sport. But there was also some insistence that we consider the question of making it relevant for students. To focus our conversation some conventional responses, which were taken from the research literature, were shared with the working group.

The NCTM Vision for Mathematics Education is a perspective offered by the profession itself. The NCTM asserts the relevance of mathematics has a number of dimensions:

- Mathematics for life: Knowing mathematics can be personally satisfying and empowering.
- Mathematics as a part of cultural heritage: Mathematics is one of the greatest cultural and intellectual achievements of humankind.
- Mathematics for the workplace: [there has been a dramatic increase in] the level of mathematical thinking and problem solving needed in the workplace, [and] in professional areas ranging from health care to graphic design.
- Mathematics for the scientific and technical community. Although all careers require a foundation of mathematical knowledge, some are mathematics intensive (NCTM, 2000, p. 4).

Ministries of Education offer a compatible version of the relevance of mathematics. Consider for example, Alberta Education's statement on Mathematics:

- Mathematic is a common human activity, increasing in importance in a rapidly advancing technological society. A greater proficiency in using mathematics increases the opportunities available to individuals. Students need to become mathematically literate in order to explore problem-solving situations, accommodate changing conditions, and actively create new knowledge in striving for self-fulfilment (AB Education, 1996, p.2).

Lynn Steen (1999) has written extensively on numeracy or quantitative literacy. Underpinning his work is the question of relevance of mathematics in society and the need for citizens to study mathematics. He classifies relevance into five categories :

- Practical: the mathematics needed for routine tasks of daily life (benefits the individual)
- Civic: the mathematics needed to understand societal issues, GNP, acid rain, globalization (benefits society)
- Professional: the mathematics skills required for work, particular jobs
- Leisure: the mathematics used for puzzles, gaming, wagering
- Cultural: the mathematics that helps us understand human culture and provides aesthetic experiences

Ole Skovsmose (2000) researches the formatting power of mathematics within society. He writes that:

> mathematics applied, for instance, in a business does not consist of 'pictures' of reality which exists prior to and independent of the modeling process. Mathematical models of advertising, marketing, investments, etc. become part of the economic reality themselves. They serve as a basis for decision-making and for economic transactions. In this way, mathematics has become part of the economic reality. This not only applies to business

> but to economic policy-making in general and not only to economy, but to categories like time, space, communication, transport, war (2000, p.4).

Skovsmose argues that school mathematics needs to do more than simply teach students to think quantitatively or statistically; it must educate our youth, our citizens, so that they begin to understand and critique the formative power of mathematics in society.

In a paper by Simmt (2001), she asked that we explore the implications of our practices. In school mathematics we often find mathematics treated in very particular ways which lead to some interesting perceptions of mathematics. Dominating student, and indeed public perception of mathematics includes views like:

- Mathematics as a set of facts, skills and procedures
- Mathematics as facts and fact
- Mathematics as either right or wrong
- Mathematics is hard

Why these perceptions and not others? Why doesn't public perception commonly include views of mathematics as a way of understanding the world in which we live or mathematics is pleasurable? In that paper, Simmt suggests that the ways in which we teach have a significant impact on how mathematics becomes relevant (or not) for society. We can educate for full participation in society or we can educate so that our mathematics has little value outside of the classroom. In her view this has more to do with how we approach the teaching and learning of mathematics than what mathematics we teach. If mathematics is taught as skills and procedures to be mastered as either right or wrong and as establishing fact and facts, then we teach a form of mathematics that has little relevance in society. If, on the other hand, we teach mathematics through investigation, variable-entry prompts[1] students have the possibility to become active and full participants in the mathematics class. Students need to be occasioned to pose problems not just solve them, to specify what is relevant rather than be told. This is the hallmark of cognition itself (Varela, Thompson and Rosch, 1999). In addition to variable-entry prompts and investigations, the demand for explanation is central. Explanation is a skill that extends well outside of the classroom and across content. Mathematics is an important site to teach students that not only is there a need to explain and that explanation must be framed in terms the "other" can understand but that an explanation is only accepted when it fits within a set of criteria (for example, axiom systems).

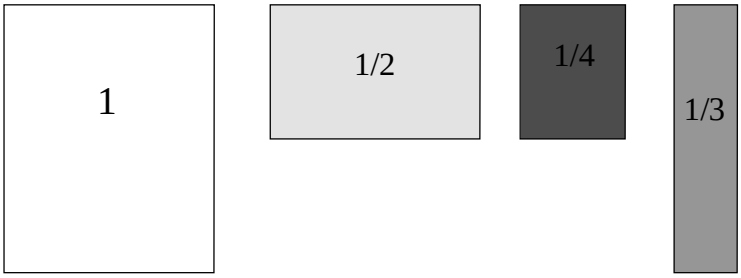
Mathematics as a common human activity, mathematics for citizenship, mathematics for the world of work… The literature cited above indicates that there are multiple answers to the question of relevancy, but what is the mathematics educator's role in this? We are left with the question of what can we do to transform the commonly espoused beliefs about mathematics, the ways in which mathematics is experienced by people and to encourage them to consider mathematical evidence, interpretations, solutions of the phenomena and problems they encounter in their day to day lives.

Participants of the working group added to the session by expressing their views on using applications from science and technology to encourage relevance.

The first session closed with an illustration from fractions of how we might teach differently to encourage meaning making, problem-posing and full participation. The example

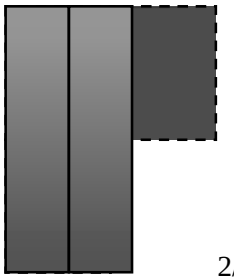
[1] These prompts have the feature that there are many different access points into the mathematics. Highly specific knowledge or previous experiences are not needed to enter into appropriate mathematical activity for the task.

uses a kit (figure 1) consisting of a variety of fractional pieces and the simple question of "I am thinking of a mystery fraction between 1/2 and 3/4. What might it be?" This question has the feature that it is variable-entry, it encourages multiple responses, it leads to addition and subtraction of fractions, the solutions initially fit within the physical constraints of the kit but can be extended through an understanding of the mathematical constraints to move beyond the possibilities affording by the kits.



The kits also included sixths, eighths, twelfths, and twenty-fourths.

A possible solution to the prompt within the constraint of the kits might be:



2/3 is greater than 1/2 but less than 3/4.

Students often move beyond the constraints of the kit in this task as they challenge themselves to create a unique solution. For example, Tom Kieren, while working with Grade 3 students has seen, 1/2 + 1/8 + 1/16 + 1/32.

One member of the working group had also brought student work to share with the group. In that work, students created posters to illustrate their understanding of fractions. This work encouraged students to look outside of the classroom for making meaning of fractions. The student posters illustrated a range of understanding and how students interpreted their world mathematically.

Day 2

CHOSES RARES OU CHOSES BELLES
ICI SAVAMMENT ASSEMBLÉES
INSTRUISENT L=OEIL À REGARDER
COME JAMAIS ENCORE VUES
TOUTES CHOSES QUI SONT AU MONDE

Rare or beautiful objects
intelligently organized here
teach the eye to see
as though never seen before
every object in the world.
Paul Valery

Day Two centred around the intimate relationship between relevance and aesthetics. Students are held by what is beautiful, often more than by what is applicable. Actually the truth is deeper than

that—the beauty of a configuration is often a good guide to its utility. In short, beauty is function, function beauty. This key principle might go a long way to explain our fundamental attraction to beauty. It is certainly not sufficiently taken account of by those who claim to design "relevant" curricula.

The truth of this can in part be seen in our evident failure to "engage" our students in the university classroom. This is documented in the 2004 study NSSE study (The National Survey of Student Engagement) developed at the University of Alberta.

Peter illustrated these ideas with "the 3-hat problem," a wonderful puzzle that never fails to capture ones attention, but which also provides the key idea behind error-correcting codes. The class played with the problem and came up with the optimal strategy. The problem is this.

Three players are sitting around a circle and either a red or a blue hat is placed on each person's head. The colour of each hat is determined by a coin toss, with the outcome of each toss having no effect on the others. Each person can see the colour of the others' hats, but not her own.

After a brief pause, during which time players study the other hats, each player either "passes" or attempts to guess the colour of the hat on her head. All three responses are simultaneous so that no player can use the information gained from the response of another player. If at least one player has guessed the colour of the hat on her head *and no player has guessed wrong* the group shares a prize of one million dollars. Otherwise, if all have passed or at least one player has guessed wrong, there is no prize.

Now the point is that these three players are not in competition; they are a team. They are not allowed to communicate with one another during the hat ceremony but they can get together before-hand and talk strategy. The question is, how well can they do? Find a strategy which will maximize the probability of winning the prize, and find that probability.

Of course this is a carefully chosen problem, and it is no surprise therefore that it turns out to be both beautiful and of great utility. But in fact there are many such examples. Theoretical physicists often say how the beauty of the theory or the configuration is the best guide to its truth.

In the discussion, a number of points were made. In many jurisdictions, courses are strongly articulated and their operative description consists mainly of a list of technical topics. Teachers might often have the desire to work with beautiful or compelling problems but feel they do not have the freedom to "stray" from the mandated path. The problem is compounded by the fact that traditional textbooks are also reluctant to seize this freedom. Paradoxically enough, the display of such freedoms can play a central role for students to become independent.

This raises the question of what exactly are the central ideas in algebra and geometry. And those questions need to be considered in the context of the mathematics class today – both in schools and in post-secondary institutions.

Day 3

Day three was set aside for a broad discussion around the central issue of relevancy of mathematics in society. It was clear that there were diverse opinions on the question that framed the working group. In the next section we offer a sample of the conversation that emerged from acting on the question of the relevance of mathematics in society and what we might do if we had sufficient resources. In order to reflect the spirit of the conversation, we group the points made into themes without attempting to link them together with further commentary.

Public perception

- A primary concern is public perception. Therefore we must ask who is our audience? Society, not mathematicians, needs to be the concern of this working group.
- There exists an attitude of math is needed but I don't need it. Everybody else but me needs math.
- For a great many in the public, mathematics is a black box, a foreign land.
- How do I convince others that it is a tool to look at and understand the world; it is a language to explain and to communicate; it is a domain in which you think.

Society and the needs of it and its members

- We need to provide opportunities at different levels not just in within our own group. We need to build understanding across groups.
- This group is about relevance. This itself is a relevant notion. Who are the stake-holders? There are multiple realities present in this topic? What is the place of mathematics in the world? We need to break down barriers that exist between views of relevance and encourage a collective view of relevance, that is a common view within our community.
- This brings about questions of relevance and accessibility.
- Think back to Eric's comments about the place of Latin in school curriculum. We need people doing math. It permeates occur society. Will only a very few people need to do math? Can we articulate a reason why we do math?
- They need it for the job market, to read the daily news, in their daily activity, in their work, and in other areas. We need an informed public.
- Teachers need concrete activities for children. We could conceive of these but then they need to spread. Maybe this could be done through teacher education?
- We need to broadened society's attitudes as to why mathematics is relevant? Broaden our own understandings.

What mathematics? Whose mathematics?

- Whose math is this? Whose society? Are we ignoring the mathematics of others? We need also to consider the mathematical understandings outside of Western ones.
- We need to understand the student and what motivates her.
- Need a multiplicity of approaches.
- We need to infuse more philosophical, historical, cultural and global dimensions.
- Mathematic is a human endeavour.

Who is responsible for addressing questions of relevance?

- We need to be champions of education.
- Responsibility for relevance does not lie only with the teacher. There is a collective responsibility and sense of relevance, a public sense of relevance.
- There are multiple stake-holders: education and other areas that students come in contact with.

What can we do?

- How do we make it relevant and accessible?
- In our teaching how do we make the case that mathematics is relevant? If we look around can we find things in the public, in society to use to build relevance? How can we as a community (CMS) work together?
- The popular press is full of interesting problems.

What are some potential outcomes of the CMS Forum?

- CMS sharing project
- Build better relationships between Faculties of Education and Departments of Mathematics.
- Create partnerships between math and math education to offer teacher pre-service and teacher in-service work.
- Create an on-line forum (e.g. WebCT, CTC) in which members of society could pose questions. We could disseminate problems, activities, worksheets through the on-line forum.
- A website consisting of math applications and models for practical things might be created.
- Statistics – a lot can be done with statistics. There is the possibility for critical thinking—to ask, where do the numbers come from? What do models for the data look like? What assumptions are there?
- Use context—can be used to capture attention and then lead to beautiful mathematics.
- Make connections—what relevance does mathematics have? These open new ways of looking at the world.
- What about the use of posters in a public campaign (potter, dancer)?
- Create a National Math Fair.
- Olympics 2010, this could be a site for a public awareness campaign or a curriculum campaign.

On the last day the group came to a consensus that **the CMS could take on curriculum projects and public awareness campaigns around national events and/or issues**. A variety of topics came up in the discussion: mathematics and social justice; mathematics and the environment; mathematics and peace; mathematics and diversity; mathematics and the 2010 Olympics; mathematics in the newspaper. Although a number of the people in the group felt mathematics and the 2010 Olympics was a timely and highly visible national event there were a few people who supported mathematics and the environment or mathematics and

social justice as the initial focus of a yet to be formed CMS mathematics in society curriculum committee.

As leaders of the CMS working group on the relevance of mathematics in society we would like to thank the members of our working group for their sustained interested in the topic over the three days. We feel we learnt more about this important topic and would strongly encourage CMS to seriously consider the recommendation put forward by this group and stated below.

We the members of the Working Group, The Relevance of Mathematics in Society, recommend: The CMS Forum initiate a process to develop curriculum projects around national issues and events. Potential issues and events include: 2010 Winter Olympics, social justice, and the environment (Kyoto) and sustainability. Curricular materials could include: modules for schools, posters, and problems for a newspaper column or other media.

The question arose as to who might be a good person to take on the leadership of the development of these projects and the name of Jean-Marie De Koninck of Universite Laval came up. Jean-Marie has a wealth of experience already with the public and with the media and would be ideal. It was agreed that Peter Taylor would write to him and sound him out on the question of developing a national project around the 2010 Winter Olympics.

Note added: Peter did write to Jean-Marie, and subsequently Jean-Marie had a conversation with Christiane Rousseau. Jean-Marie's opinion was that it is maybe premature to start a pan-Canadian event around the Olympics and that the connection with mathematics has to be very well prepared before any movement with the media is made. So in the short term he would prefer to wait and see what kind of events the teachers of BC are preparing. He is of course willing to help for the contacts with the medias. Jean-Marie was much more enthusiastic with a theme around environmental issues. Indeed the public is sensitive to that, and it should be easier to find money. So Jean-Marie would favour starting activities around this theme and postpone for a while activities around the Olympics.

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