Report of Working Group 3: Popularisation of mathematics

The working group was organised collaboratively by Kathy Heinrich, Ron Lancaster, Ralph Mason and David Reid. Participants included Ivar Ekeland, Sara Johnson, Joyce Millard and John Grant McLoughlin. The report is a combined effort of all the participants.

Our first session began with presentations by Ron and Kathy of examples of the kinds of mathematical resources and activities they saw as having the potential to popularise mathematics.

Ron recommended two books, *In Code* by Sarah Flannery, and *The Number Devil* by Hans Magnus Enzensberger, as examples of the kinds of resources that are readily available to teachers looking for inspiration for more engaging and humanised mathematics teaching. The fact that these books were not familiar to most of us lead to the first of many questions raised: "How can mathematics teachers, and teacher educators become more aware of useful resources that already exist?"

Ron next introduced us to a mathematical activity:

> Beginning with four numbers (e.g., 5 7 11 8) write below each of them the difference between the number and its neighbour to the right. In the case of the right most number write the difference between it and the left most number. In all cases take the positive difference. Repeat this rule. Observe what happens. Explain why.
>
> Example:

5	7	11	8
2	4	3	3
2	1	0	1
1	1	1	1
0	0	0	0

This activity possesses many features that we saw as important for a popularising activity.

- It is immediately accessible to the intended audience. All that is required is an ability to subtract to begin exploring.
- There is an opportunity to engage in mathematical reasoning, searching for an explanation for the convergence to 0.
- The problem immediately leads to others: What is the effect of changing one or more of the starting numbers? How can the number of steps before arriving at 0 be made longer? What happens if you start with a different number of numbers?
- There is a surprising element in the activity. The convergence to 0 is likely to surprise some people. Examining the behaviour of geometric sequences leads to further surprises in case the initial surprise in not enough.
- There are opportunities to extend the problem to include a range of mathematical explorations, one of which Ron described in our final session (see below).

Our discussion of this activity also raised several important issues around the popularization of mathematics, to which we returned in later sessions. These included:

- The need to "cover" the curriculum leaving little time for explorations like this which touch on mathematical topics that might not all occur in the list of topics for a single grade.
- The structure of curriculum documents around topics, instead of, as has been sometimes proposed, around interesting problems.
- The overwhelming number of such problems and the difficulty for teachers of knowing what problems might be popular with their students.
- The challenge for teachers in identifying the right problems and then "making the case" that working on them will allow for coverage of significant parts of the curriculum.

Kathy then offered another problem for discussion:

> Does there exist an irrational number *a* which when raised to an irrational power *b* is rational?

This led us into a discussion of "nice" proofs in mathematics, and the more general issue of the dependence of what is a nice proof or a nice problem on the knowledge and experience of the audience. A problem that we find interesting as mathematicians might not be as interesting to a classroom teacher faced with more immediate problems, and a problem of interest to teachers might not be of interest to students. We recognized that **Context** is vital to making a problem interesting.

We tried to identify some words that describe problems and proofs that are interesting to us as mathematicians. These included: Unexpected, symmetry, simple, beautiful, transparent, visual, how the proof unfolds before your eyes. This raised the questions of why everyone does not seem to find these characteristics as interesting as we do.

Some descriptions of activities that we have found students find interesting were then proposed: No pressure, anyone can succeed, not intimidating, multiple solutions, rewards for staying with it.

Ralph then reminded us of how much of our understanding and appreciation of mathematics depends on unconscious judgments. As an example he reminded us of the standard "proof" of the equality 0.999999… = 1:

$10x = 9.9999999\ldots$
$x = 0.9999999\ldots.$
so $9x = 9$, so $x = 1$

He contrasted this with the following "proof":

$2x = 2 + 4 + 8 + \ldots$
$x = 1 + 2 + 4 + 8 + \ldots$
so $x = -1$

We recognize that there is something wrong here. How can students come to share this feeling?

In the second session Ralph offered another contrast, between a Jeopardy style review game for mathematical facts and a game played on a board listing numbers from 1 to *n* in which the players alternate eliminating numbers, but can only eliminate a number which is a multiple or factor of the previous player's choice. This provided us with an illustration of the difference between "dressing up" mathematics (as a jeopardy game for example) and building on interesting aspects of mathematics (the central importance of prime numbers for example) to create an interesting activity.
Ralph proposed six aspects of an activity related to making it interesting, not all of which would be equally important in all cases: Context, Curiosity, Content, Success, Interaction, and Action.

In the third session we returned to the issues raised in the first two sessions and tried to structure our work around some questions and responses to them:

What makes students find mathematics interesting?

- Context matters. Calling something school math automatically makes it less interesting to some.
- "No pressure, anyone can succeed. Not intimidated by what you are proposing."
- "Multiple solutions. Reward for staying with it."
- Can anything be compelling, if we've made it compulsory?
- Context, Curiosity, Content, Success, Interaction, Action

What barriers are preventing teachers from teaching a more popular mathematics?

- Over specified curricula.
- Lack of information about good activities and books
- Time
- Experience and confidence

What would a curriculum supporting a more popular mathematics be like?

- Problems instead of outcomes?
- Intrinsic rewards instead of extrinsic punishments?

How are problems/activities related to outcomes? Can we teach problems/activities within existing curricula?

- Would it be possible to take a problem or activity we have seen and specify outcomes for it?

What would we recommend to ministries of education as an action they (we) could take to popularise mathematics?

- Share the teaching of the teachers who are already actively involved in mathematics exploration through problem solving.

What would we recommend to mathematics departments as an action they (we) could take to popularise mathematics?

- Transform undergraduate mathematics by including more popular activities; for example, a general course on mathematics that covers a variety of accessible problems/projects that are engaging and challenging.

Recommendation for work before the next forum:

The creation, by the members of the group, of a model or models of a problem based activity that includes the background information (curriculum links, extensions, possible solution paths, etc.) that would make a teacher's use of the activity less difficult.

An examination of models that would help teachers to do this for themselves.

Possible long term projects related to the popularization of mathematics:

- Analysis and cataloguing of good problems in a format accessible to teachers (following the model(s) developed above).
- Pushing for curriculum change, towards a more problem based approach