Teaching geometry effectively with dynamic software

Report by Margaret Sinclair, York University

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Abstract
This workshop examined issues related to the effective teaching of geometry with dynamic geometry software. The discussions mainly focused on the development and use of curriculum-related sketches that are either partially constructed, or pre-constructed. Examples of web-based and program-based sketches created with Geometer's Sketchpad and Cinderella were presented and discussed in light of research results and the experience of participants. An important theme throughout the discussions was how to address the needs of teachers – to develop greater geometric knowledge, to access well-designed materials, and to acquire technological skills.

In light of the theme of critical thinking we began by examining how pre-constructed dynamic geometry sketches can help students think about and analyse mathematical ideas. In our definition of pre-constructed sketches we included both web-based and program-based dynamic geometry sketches, whether prepared by teachers, by researchers, or by professional applet designers. Some of these could more properly be named “partially-constructed”, since students are able to alter any program-based sketch and some web-based sketches.

Sinclair presented some results from a study that involved grade 12 students using pre-constructed web-based sketches. Her research showed that to benefit from using dynamic pre-constructed sketches, students must learn: a) to notice (i.e., to notice detail in static images, to notice change in dynamic images and to apply mathematical knowledge to make sense of the information); b) to pose (their own) questions; and c) to explore. The development of these skills depends both on pedagogical interventions and on the creation of appropriate learning objects.

With regard to pedagogy, teachers who want to use dynamic software activities face new challenges. For example, there is a great deal of talk about “investigations” in mathematics classes; however, unlike investigators who continually re-evaluate their progress, and determine how to proceed, students often rely on the teacher to suggest the next step or ask the question that will guide further exploration. Sinclair found that students working in pairs in a lab environment are often unable to engage in fruitful ongoing investigation if they cannot pose their own questions. We discussed the importance of helping inservice and preservice teachers learn how to make effective use of dynamic software tasks.

Good design decisions (i.e., what colours to use; whether to display tick marks or measurements; whether to animate) are intimately related to knowledge of what helps us

notice. Sinclair found that colour and movement helped students notice details, but that onscreen measurements drew little attention. Particular onscreen or labsheet prompts helped students explore, while others were ineffective. Animations or motion buttons gave some students an idea of how to investigate.

A discussion about sketches/applets and lessons that incorporate them ensued. The difficulties of preparing good sketches -- both from a geometric and from a technological standpoint – and the need to help teachers share/access good models led to an examination of the idea of a learning object repository. It was noted that the Merlot Learning Object Repository http://www.merlot.org/Home.po is mainly devoted to university teaching applications but that a new digital library called MathTools http://mathforum.org/mathtools/, which will address the needs of elementary, secondary and college teachers has recently been set up at The Math Forum. Both of these initiatives are focused on American curricula. A new learning objects repository is being developed at Brock university http://www.brocku.ca/learningobjects/flash_content/index.html. The creation of several learning objects related to the Ontario high school mathematics curriculum is presently underway.

When students are able to make sense of a dynamic image, they are then able to use it to develop and test conjectures. From the teacher's perspective, a pre-constructed sketch can act like a carefully chosen problem that uncovers particular mathematical relationships and features. In addition, when a sketch is pre-constructed, students are able to spend more of their class time investigating and conjecturing, which should result in more profitable class discussions. Ysbrand de Bruyn's presentation illustrated both of these ideas. De Bruyn chose an activity that involved a unique investigation related to theorems on inscribed angles, and angles formed by secants. He noted that the investigation holds the interest of his students and helps them develop their conjecturing, and proving skills. De Bruyn demonstrated the activity using Geometers' Sketchpad, but also discussed the applicability of Cinderella. Although Cinderella is not available in Ontario schools, it can be used to prepare web-ready sketches that can be altered by means of one or more online tools; however, both Sinclair and de Bruyn admitted that (in the present version of Cinderella) providing the tools is not simple!

The discussion that followed centred on issues related to sketch design in the Sketchpad and Cinderella environments. It was generally agreed that creating superior sketches is a difficult task and that teachers need access to well-designed sketches in order to successfully incorporate dynamic software activities into their programs. But it was also noted that teachers' choices for activities are constrained by the requirements of their curriculum; this makes it important to develop sketches and lesson plans that address curriculum topics – i.e., not just enrichment mathematics.

We briefly discussed the challenge of helping teachers and teacher candidates to use dynamic software. Inservice seminars on dynamic geometry software, while available, are often of little use if teachers cannot depend on access to hardware. Another obstacle is that teachers and preservice candidates have often had impoverished geometry experiences. For many, geometry means knowing the names of shapes, being able to

identify transformations, and using ITT and the congruency theorems in a 2-column proof. It was noted that preservice mathematics courses are too short to address with sufficient depth both mathematics concepts and the methods of teaching.

At the third session, Nick Jackiw joined us. He demonstrated several new features of Sketchpad 4, and discussed some of the ways in which having students construct their own diagrams is important in helping them develop geometric thinking skills. He has reservations about the use of pre-constructed sketches that only allow students to drag points; however, he agrees that pre-constructed and "partially constructed" sketches can provide interesting opportunities for learning. For example, he demonstrated an application that allows students to predict and then check the appearance of a shape in a one dimensional world – a wonderful interactive object to help students develop their visual and spatial reasoning skills.

During the sessions we also looked at Spherical Sketchpad (no connection to the Geometer's Sketchpad) a new web-based application. The discussion focused on the intuitive nature of the interface and possible applications of Spherical Sketchpad, e.g., as a tool for enrichment at the high school level or to broaden teachers' conception of geometry in inservice or graduate courses.

Recommendations

The group did not formally state recommendations; however, some of the discussions suggest the following as possible goals for the next two years:

In the area of sketch development and use:

- Create curriculum-specific sketches and lesson plans;
- Support the development of learning object repositories – especially those that address the Canadian context (e.g., Brock University); in conjunction, consider a peer-review process for these learning objects;
- Continue to research the features of pre-constructed or partially-constructed dynamic geometry sketches/applets that support student learning.

In the area of teacher education:

- Recommend that all preservice programs include an introduction to the use of dynamic geometry software;
- Emphasise the importance for teachers of developing enhanced geometry skills, especially those related to visual and spatial understanding;
- Publicize learning object repositories.

At the next forum, we hope that there will be an opportunity to examine learning object repositories in Canada – to judge the caliber of the objects themselves, and to gauge whether or how sharing geometry sketches and lesson plans has affected the teaching of geometry.

We also hope that by the next forum, preservice mathematics courses will have been expanded to include greater emphasis on visual and spatial skills, and an introduction to the use of dynamic software.