Report of the Thirty Ninth Canadian Mathematical Olympiad 2007

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The Canadian Mathematical Olympiad (CMO) is an annual national mathematics competition sponsored by the Canadian Mathematical Society (CMS) and is administered by the Canadian Mathematical Olympiad Committee (CMO Committee), a sub-committee of the Mathematical Competitions Committee. The CMO was established in 1969 to provide an opportunity for students who performed well in various provincial mathematics competitions to compete at a national level. It also serves as preparation for those Canadian students competing at the International Mathematical Olympiad (IMO).

Students qualify to write the CMO by earning a sufficiently high score on the Canadian Open Mathematical Challenge (COMC). Other students may be nominated to write the CMO at the discretion of the Chair. I am indebted to Peter Minev of the University of Alberta for nominating additional students from the Province of Alberta.

The Society is grateful for support from the **Sun Life Assurance Company of Canada** as sponsor of the 2006 Canadian Mathematical Olympiad and the other sponsors which include: the Ministry of Education of Ontario; the Ministry of Education of Quebec; Alberta Learning; the Department of Education, New Brunswick; the Department of Education, Newfoundland and Labrador; the Department of Education, the Northwest Territories; the Department of Education of Saskatchewan; the Department of Mathematics and Statistics, University of New Brunswick at Fredericton; the Centre for Education in Mathematics and Computing, University of Waterloo; the Department of Mathematics and Statistics, University of Ottawa; the Department of Mathematics, University of Toronto; the Department of Mathematics, University of British Columbia; Nelson Thompson Learning; John Wiley and Sons Canada Ltd.; McGraw-Hill; A.K. Peters and Maplesoft.

I am very grateful to the CMO Committee members who submitted problems that were considered for the 2007 competition: Robert Barrington Leigh, Ed Doolittle, Chris Fisher, Valeria Pendelieva, Naoki Sato and Jacob Tsimerman. [I note with great sorrow and regret the death of Robert Barrington Leigh in August, 2006.] The papers were marked by Ed Barbeau, Man-Duen Choi, Felix Recio and Adrian Tang, with assistance on particular solutions from Chris Fisher, Naoki Sato, Adrian Tang and Ed Wang. Thanks to Tom Griffiths of London, ON for validating the paper and to Joseph Khoury for translating the paper into French. I am indebted for the hard work done at CMS headquarters by Susan Latreille and the Executive Director, Graham Wright, whose continued commitment is a vital ingredient of the success of the CMO.

Ed Barbeau, Chair
Canadian Mathematical Olympiad Committee

The 39th (2007) Canadian Mathematical Olympiad was held on Wednesday, March 28, 2007. A total of 76 students from 50 schools (46 in Canada, three in the US and one in Hong Kong) wrote the paper. Seven Canadian provinces were represented, with the number of contestants as follows:

BC (11) AB (6) SK (1) ON (43) QC (2) NB (1) NS (1)

The 2007 CMO consisted of five questions, each marked out of 7. The maximum score attained by a student was 30. The official contestants were grouped into four divisions according to their scores as follows:

Division	Range of Scores	No. of Students
I	23 - 30	7
II	19 - 22	14
III	15 - 18	21
IV	0 - 14	33

FIRST PRIZE — Sun Life Financial Cup — $2000

Yan Li
Dr. Norman Bethune Collegiate Institute, Scarborough, Ontario

SECOND PRIZE — $1500
Jonathan Schneider
University of Toronto Schools, Toronto, Ontario

THIRD PRIZE — $1000
Jarno Sun
Western Canada High School, Calgary, Alberta

HONOURABLE MENTIONS — $500

Jia Guo
O'Neill Collegiate Vocational Institute
Oshawa, ON

Kent Huynh
University of Toronto Schools
Toronto, ON

Steven Karp
Lord Byng Secondary School.
Vancouver, BC

Alexander Remorov
William Lyon Mackenzie Collegiate Institute
North York, ON

Division 1

23 - 30

Yan Li	Dr. Norman Bethune C.I.	ON
Jonathan Schneider	University of Toronto Schools	ON
Jarno Sun	Western Canada H.S.	AB
Jeffrey Mo*	William Aberhart H.S.	AB
Alexander Remorov	William Lyon Mackenzie C.I.	ON
Steven Karp	Lord Byng S.S.	BC
Jia Guo	O'Neill C.V.I.	ON
Kent Huynh	University of Toronto Schools	ON

Division 2

19 - 22

Jimmy He	Seaquam S.S.	BC
Chen Sun	A.B. Lucas S.S.	ON
Linda Zhang	Western Canada H.S.	AB
Lin Fei	Don Mills C.I.	ON
Greg Tsang	Crescent School	ON
Haolong Zheng	London Int'l Academy	ON
William Fu	A.Y. Jackson S.S.	ON
Frank Meng	Burnaby South S.S.	BC
Danny Shi	Sir Winston Churchill H. S.	AB
Bobby Xiao	Walter Murray C.I.	SK
Sunil Argarwal	ICAE	MI
Bo Cheng Cui	West Vancouver S.S.	BC
Xiao Jiang	Marianopolis College	QC
Gary Peng	Don Mills C.I.	ON

Division 3

15 - 18

Bill Long	Glebe Collegiate Institute	ON
Johnson Mo	St. George's School	BC
Julian Sun	A.B. Lucas S.S.	ON
Arman Tavakoli	Kitsilano Secondary School	BC
Chen Bo Wan	Stephen Leacock C.I.	ON
Catherine Zhou	Albert Campbell C.I.	ON
Philip Chen	Glenforest S.S.	ON
Joe Kileel	Fredericton H.S.	NB
Kyle MJ Kim	Unionville H.S.	ON
Yifan Wang	Laura Secord S.S.	ON
Hao Yan	Jarvis C.I.	ON
Andy Kong	Vincent Massey S.S.	ON
Sina Makaremi	John Fraser S.S.	ON
Jonathan Zhou	Burnaby North Secondary School	BC
Neil Gurram	ICAE	MI
Jiayang Jiang	A.Y. Jackson S.S.	ON
Sheng Liu	Stephen Leacock C.I.	ON
David Wang	A.B. Lucas S.S.	ON
Shuyun Wu	Martingrove C.I.	ON
Chung Liang Yee	St. Paul's Co-educational College	HK
Tianyuan Zheng	Phillips Academy	MA

Division 4

0 - 14

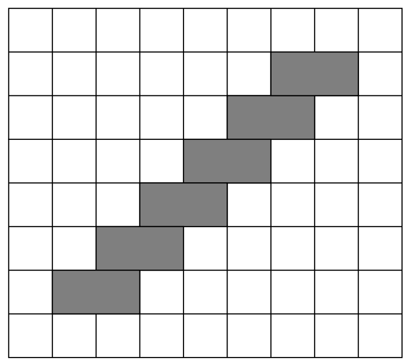
Boris Braverman	Sir Winston Churchill H. S.	AB
Chuan Guo	St. Pius X High School	ON
Zhiqiang Liu	Don Mills C.I.	ON
Benjamin Niedzielski	Phillips Academy	MA
Max Zhou	L'Amoreaux C.I.	ON
Harry Chang	A.B. Lucas S.S.	ON
Simeng Ding	Waterloo C.I.	ON
Dimitri Dziabenko	Don Mills C.I.	ON
Kwonyong Jin	Phillips Academy	MA
Steven Wu	Martingrove C.I.	ON
Terry Zhang	Sir John A. Macdonald C.I.	ON
Hao Chen	The Woodlands S.	ON
Saurabh Pandey	ICAE	
Zhang Xinyang	Orillia D.C. & V.I.	ON
Shi Yao Zhang	Jarvis C.I.	ON
Corey Yednoroz	Vincent Massey S.S.	ON
Yunoso Kim	Phillips Academy	MA
Yang Zhou	Albert Campbell C.I.	ON
Fan Jiang	Albert Campbell C.I.	ON
James Yang	Phillips Academy	MA
Frank Ban	Vincent Massey S.S.	ON
Ram Bhaskar	ICAE	MI
Juliet Ji	Georges Vanier S.S.	ON
Heesung Yang	West Vancouver S.S.	BC
Bill Pang	Sir Winston Churchill S.S.	BC
Hwi Lee	Gleneagle S.S.	BC
Linhe Li	Queen Elizabeth H.S.	NS
Michael Wong	Tempo School	AB
Kou Kou	Bond Academy	ON
Ran Li	E.S. Honore-Mercier	QC
Takwai Lui	St. Paul's Co-educational College	HK
Vincent Zhou	Dr. Norman Bethune C.I.	ON
Daniel Lee	Crescent School	ON

*unofficial candidate

39th Canadian Mathematical Olympiad

Wednesday, March 28, 2007

1. What is the maximum number of non-overlapping 2×1 dominoes that can be placed on an 8×9 checkerboard if six of them are placed as shown? Each domino must be placed horizontally or vertically so as to cover two adjacent squares of the board.



2. You are given a pair of triangles for which

   (a) two sides of one triangle are equal in length to two sides of the second triangle, and

   (b) the triangles are similar, but not necessarily congruent.

   Prove that the ratio of the sides that correspond under the similarity is a number between $\frac{1}{2}(\sqrt{5}-1)$ and $\frac{1}{2}(\sqrt{5}+1)$.

3. Suppose that f is a real-valued function for which

$$f(xy) + f(y - x) \geq f(y + x)$$

   for all real numbers x and y.

   (a) Give a nonconstant polynomial that satisfies the condition.

   (b) Prove that $f(x) \geq 0$ for all real x.

Go to page 2 for the remaining questions

4. For two real numbers a, b, with $ab \neq 1$, define the $*$ operation by

$$a * b = \frac{a + b - 2ab}{1 - ab} \ .$$

Start with a list of $n \geq 2$ real numbers whose entries x all satisfy $0 < x < 1$. Select any two numbers a and b in the list; remove them and put the number $a * b$ at the end of the list, thereby reducing its length by one. Repeat this procedure until a single number remains.

(a) Prove that this single number is the same regardless of the choice of pair at each stage.

(b) Suppose that the condition on the numbers x in S is weakened to $0 < x \leq 1$. What happens if S contains exactly one 1?

5. Let the incircle of triangle ABC touch sides BC, CA and AB at D, E and F, respectively. Let Γ, Γ_1, Γ_2 and Γ_3 denote the circumcircles of triangle ABC, AEF, BDF and CDE respectively.

Let Γ and Γ_1 intersect at A and P, Γ and Γ_2 intersect at B and Q, and Γ and Γ_3 intersect at C and R.

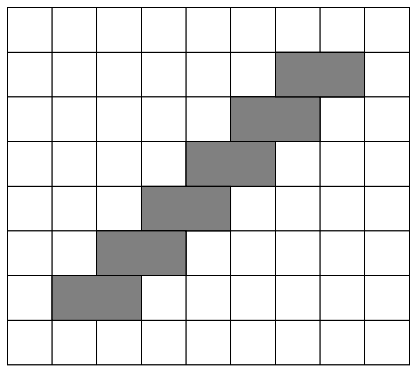
(a) Prove that the circles Γ_1, Γ_2 and Γ_3 intersect in a common point.

(b) Show that PD, QE and RF are concurrent.

39th Canadian Mathematical Olympiad

Wednesday, March 29, 2007

Solutions to the 2007 CMO paper



Solution to 1. Identify five subsets A, B, C, D, E of the board, where C consists of the squares occupied by the six dominos already placed, B is the upper right corner, D is the lower left corner, A consists of the squares above and to the left of those in $B \cup C \cup D$ and E consists of the squares below and to the right of those in $B \cup C \cup D$. The board can be coloured checkerboard fashion so that A has 13 black and 16 white squares, B a single white square, E 16 black and 13 white squares and D a single black square. Each domino beyond the original six must lie either entirely in $A \cup B \cup D$ or $C \cup B \cup D$, either of which contains at most 14 dominos. Thus, altogether, we cannot have more that $2 \times 14 + 6 = 34$ dominos. This is achievable, by placing 14 dominos in $A \cup D$ and 14 in $E \cup B$.

Solution to 2. If the triangles are isosceles, then they must be congruent and the desired ratio is 1. For, if they share equal side lengths, at least one of these side lengths on one triangle corresponds to the same length on the other. And if they share unequal side lengths, then either equal sides correspond or unequal sides correspond in both directions and the ratio is 1. This falls within the bounds.

Let the triangles be scalene. It is not possible for the same length to be an extreme length (largest or smallest) of both triangles. Therefore, we must have a situation in which the corresponding side lengths of the two triangles are (x, y, z) and (y, z, u) with $x < y < z$ and $y < z < u$. We are given that $y/x = z/y = u/z = r > 1$. Thus, $y = rx$ and $z = ry = r^2x$. From the triangle inequality $z < x + y$, we have that $r^2 < 1 + r$. Since $r^2 - r - 1 < 0$ and $r > 1$, $1 < r < \frac{1}{2}(\sqrt{5} + 1)$. The ratio of the dimensions from the smaller to the larger triangle is $1/r$ which satisfies $\frac{1}{2}(\sqrt{5} - 1) < 1/r < 1$. The result follows.

Solution to 3. (a) Let $f(x) = x^2 + 4$. Then

$$\begin{aligned} f(xy) + f(y - x) - f(y + x) &= (x^2y^2 + 4) + (y - x)^2 + 4 - (y + x)^2 - 4 \\ &= (xy)^2 - 4xy + 4 = (xy - 2)^2 \geq 0 \ . \end{aligned} \tag{1}$$

Thus, $f(x) = x^2 + 4$ satisfies the condition.

1

(b) Consider (x, y) for which $xy = x + y$. Rewriting this as $(x-1)(y-1) = 1$, we find that this has the general solution $(x, y) = (1+t^{-1}, 1+t)$, for $t \neq 0$. Plugging this into the inequality, we get that $f(t - t^{-1}) \geq 0$ for all $t \neq 0$. For arbitrary real u, the equation $t - t^{-1} = u$ leads to the quadratic $t^2 - ut - 1 = 0$ which has a positive discriminant and so a real solution. Hence $f(u) \geq 0$ for each real u.

Comment. The substitution $v = y - x$, $u = y + x$ whose inverse is $x = \frac{1}{2}(u - v)$, $y = \frac{1}{2}(u + v)$ renders the condition as $f(\frac{1}{4}(u^2 - v^2)) + f(v) \geq f(u)$. The same strategy as in the foregoing solution leads to the choice $u = 2 + \sqrt{v^2 + 4}$ and $f(v) \geq 0$ for all v.

Solution to 4 (b). It is straightforward to verify that $a * 1 = 1$ for $a \neq 1$, so that once 1 is included in the list, it can never by removed and so the list terminates with the single value 1.

Solution to 4 (a). There are several ways of approaching (a). It is important to verify that the set $\{x : 0 < x < 1\}$ is closed under the operation $*$ so that it is always defined.

If $0 < a, b < 1$, then

$$0 < \frac{a + b - 2ab}{1 - ab} < 1 \ .$$

The left inequality follows from

$$a + b - 2ab = a(1 - b) + b(1 - a) > 0$$

and the right from

$$1 - \frac{a + b - 2ab}{1 - ab} = \frac{(1 - a)(1 - b)}{1 - ab} > 0 \ .$$

Hence, it will never happen that a set of numbers will contain a pair of reciprocals, and the operation can always be performed.

Solution 1. It can be shown by induction that any two numbers in any of the sets arise from disjoint subsets of S.

Use an induction argument on the number of entries that one starts with. At each stage the number of entries is reduced by one. If we start with n numbers, the final result is

$$\frac{\sigma_1 - 2\sigma_2 + 3\sigma_3 - \cdots + (-1)^{n-1} n \sigma_n}{1 - \sigma_2 + 2\sigma_3 - 3\sigma_4 + \cdots + (-1)^{n-1}(n-1)\sigma_n} \ ,$$

where σ_i is the symmetric sum of all $\binom{n}{i}$ $i-$fold products of the n elements x_i in the list.

Solution 2. Define

$$a * b = \frac{a + b - 2ab}{1 - ab} \ .$$

This operation is commutative and also associative:

$$a * (b * c) = (a * b) * c = \frac{a + b + c - 2(ab + bc + ca) + 3abc}{1 - (ab + bc + ca) + 2abc} \ .$$

Since the final result amounts to a $*-$product of elements of S with some arrangement of brackets, the result follows.

Solution 3. Let $\phi(x) = x/(1 - x)$ for $0 < x < 1$. This is a one-one function from the open interval $(0, 1)$ to the half line $(0, \infty)$. For any numbers $a, b \in S$, we have that

$$\begin{aligned} \phi\left(\frac{a + b - 2ab}{1 - ab}\right) &= \tfrac{a+b-2ab}{(1-ab)-(a+b-2ab)} = \tfrac{a+b-2ab}{1-a-b+ab} \\ &= \tfrac{a}{1-a} + \tfrac{b}{1-b} = \phi(a) + \phi(b) \ . \end{aligned} \tag{2}$$

Let $T = \{\phi(s) : s \in S\}$. Then replacing a, b in S as indicated corresponds to replacing $\phi(a)$ and $\phi(b)$ in T by $\phi(a) + \phi(b)$ to get a new pair of sets related by ϕ. The final result is the inverse under ϕ of $\sum\{\phi(s) : s \in S\}$.

Solution 4. Let $f(x) = (1 - x)^{-1}$ be defined for positive x unequal to 1. Then $f(x) > 1$ if and only if $0 < x < 1$. Observe that

$$f(x * y) = \frac{1 - xy}{1 - x - y + xy} = \frac{1}{1 - x} + \frac{1}{1 - y} - 1 \ .$$

2

If $f(x) > 1$ and $f(y) > 1$, then also $f(x * y) > 1$. It follows that if x and y lie in the open interval $(0, 1)$, so does $x * y$. We also note that $f(x)$ is a one-one function.

To each list L, we associate the function $g(L)$ defined by

$$g(L) = \sum \{f(x) : x \in L\} \ .$$

Let L_n be the given list, and let the subsequent lists be $L_{n-1}, L_{n-2}, \cdots, L_1$, where L_i has i elements. Since $f(x * y) = f(x) + f(y) - 1$, $g(L_i) = g(L_n) - (n - i)$ regardless of the choice that creates each list from its predecessors. Hence $g(L_1) = g(L_n) - (n - 1)$ is fixed. However, $g(L_1) = f(a)$ for some number a with $0 < a < 1$. Hence $a = f^{-1}(g(L_n) - (n - 1))$ is fixed.

Solution to 5 (a). Let I be the incentre of triangle ABC. Since the quadrilateral $AEIF$ has right angles at E and F, it is concyclic, so that Γ_1 passes through I. Similarly, Γ_2 and Γ_3 pass through I, and (a) follows.

Solution to 5 (b). Let ω and I denote the incircle and incentre of triangle ABC, respectively. Observe that, since AI bisects the angle FAE and $AF = AE$, then AI right bisects the segment FE. Similarly, BI right bisects DF and CI right bisects DE.

We invert the diagram through ω. Under this inversion, let the image of A be A', *etc.* Note that the centre I of inversion is collinear with any point and its image under the inversion. Under this inversion, the image of Γ_1 is EF, which makes A' the midpoint of EF. Similarly, B' is the midpoint of DF and C' is the midpoint of DE. Hence, Γ', the image of Γ under this inversion, is the circumcircle of triangle $A'B'C'$, which implies that Γ' is the nine-point circle of triangle DEF.

Since P is the intersection of Γ and Γ_1 other than A, P' is the intersection of Γ' and EF other than A', which means that P' is the foot of the altitude from D to EF. Similarly, Q' is the foot of the altitude from E to DF and R' is the foot of the altitude from F to DE.

Now, let X, Y and Z be the midpoints of arcs BC, AC and AB on Γ respectively. We claim that X lies on PD.

Let X' be the image of X under the inversion, so I, X and X' are collinear. But X is the midpoint of arc BC, so A, A', I, X' and X are collinear. The image of line PD is the circumcircle of triangle $P'ID$, so to prove that X lies on PD, it suffices to prove that points P', I, X' and D are concyclic.

We know that B' is the midpoint of DF, C' is the midpoint of DE and P' is the foot of the altitude from D to EF. Hence, D is the reflection of P' in $B'C'$.

Since $IA' \perp EF$, $IB' \perp DF$ and $IC' \perp DE$, I is the orthocentre of triangle $A'B'C'$. So, X' is the intersection of the altitude from A' to $B'C'$ with the circumcircle of triangle $A'B'C'$. From a wellknown fact, X' is the reflection of I in $B'C'$. This means that $B'C'$ is the perpendicular bisector of both $P'D$ and IX', so that the points P', I, X' and D are concyclic.

Hence, X lies on PD. Similarly, Y lies on QE and Z lies on RF. Thus, to prove that PD, QE and RF are concurrent, it suffices to prove that DX, EY and FZ are concurrent.

To show this, consider tangents to Γ at X, Y and Z. These are parallel to BC, AC and AB, respectively. Hence, the triangle Δ that these tangents define is homothetic to the triangle ABC. Let S be the centre of homothety. Then the homothety taking triangle ABC to Δ takes ω to Γ, and so takes D to X, E to Y and F to Z. Hence DX, EY and FZ concur at S.

Comment. The solution uses the following result: *Suppose ABC is a triangle with orthocentre H and that AH intersects BC at P and the circumcircle of ABC at D. Then $HP = PD$.* The proof is straightforward: Let BH meet AC at Q. Note that $AD \perp BC$ and $BQ \perp AC$. Since $\angle ACB = \angle ADB$,

$$\angle HBC = \angle QBC = 90^\circ - \angle QCB = 90^\circ - \angle ACB = 90^\circ - \angle ADB = \angle DBP \ ,$$

from which follows the congruence of triangle HBP and DBP and equality of HP and PD.

Solution 2. (a) Let Γ_2 and Γ_3 intersect at J. Then $BDJF$ and $CDJE$ are concyclic. We have that

$$\begin{aligned} \angle FJE &= 360^\circ - (\angle DJF + \angle DJE) \\ &= 360^\circ - (180^\circ - \angle ABC + 180^\circ - \angle ACB) \\ &= \angle ABC + \angle ACB = 180^\circ - \angle FAE \ . \end{aligned} \tag{3}$$

Hence $AFJE$ is concyclic and so the circumcircles of AEF, BDF and CED pass through J.

(b) [Y. Li] Join RE, RD, RA and RB. In Γ_3, $\angle ERD = \angle ECD = \angle ACB$ and $\angle REC = \angle RDC$. In Γ, $\angle ARB = \angle ACB$. Hence, $\angle ERD = \angle ARB \Longrightarrow \angle ARE = \angle BRD$. Also,

$$\angle AER = 180^\circ - \angle REC = 180^\circ - \angle RDC = \angle BDR \ .$$

Therefore, triangle ARE and BRD are similar, and $AR : BR = AE : BD = AF : BF$. If follows that RF bisects angle ARB, so that RF passes through the midpoint of minor arc AB on Γ. Similarly, PD and QE are respective bisectors of angles BPC and CQA and pass through the midpoints of the minor arc BC and CA on γ..

Let O be the centre of circle Γ, and U, V, W be the respective midpoints of the minor arc BC, CA, AB on this circle, so that PU contains D, QV contains E and RW contains F. It is required to prove that DU, EV and FW are concurrent.

Since ID and OU are perpendicular to BC, $ID \| OU$. Similarly, $IE \| OV$ and $IF \| OW$. Since $|ID| = |IE| = |IF| = r$ (the inradius) and $|OU| = |OV| = |OW| = R$ (the circumradius), a translation $\overrightarrow{IO}$ followed by a dilatation of factor R/r takes triangle DEF to triangle UVW, so that these triangles are similar with corresponding sides parallel.

Suppose that EV and FW intersect at K and that DU and FW intersect at L. Taking account of the similarity of the triangles KEF and KVW, LDF and LUW, DEF and UVW, we have that

$$KF : FW = EF : VW = DF : UW = LF : LW \ ,$$

so that $K = L$ and the lines DU, EV and FW intersect in a common point K, as desired.

THE GRADERS' REPORT

The grading was done by Ed Barbeau, Man-Duen Choi, Felix Recio and Adrian Tang. The top papers were reviewed in detail also by Ed Wang, and assistance in evaluating specific problems was provided by Chris Fisher, Naoki Sato and Jacob Tsimerman. The papers in the top three divisions and many of those in the fourth were independently marked by at least two people, and at least four people determined the ranking of the top dozen.

The problems were designed so that there was a part that students could do for a reasonably straightforward 2 marks. This included a specific covering with dominoes in Question 1, as well as Problems 3(a), 4(b) and 5(a). There were a surprising nmber of students who did not take advantage of this.

78 students were registered to write the CMO, but two failed to show up on the day of the competition.

The marks awarded on the several problems are given in the following table:

Marks	#1	#2	#3	#4	#5
7	24	23	5	6	1
6	5	12	1	25	0
5	10	8	1	11	1
4	4	8	2	4	0
3	4	5	5	3	0
2	16	7	23	10	39
1	2	7	6	2	3
0	6	4	23	5	4
-	5	2	10	10	28

Problem 1: Two marks were awarded for an example of a placement of 34 dominoes. Several students who had an otherwise correct argument failed to show that the maximum of 34 was indeed possible. Many students coloured the checkerboard with black and white squares and exploited the fact that a domino had to cover a square of each colour. A common strategy was to split the uncovered squares into two triangular regions and deal with each separately, although not everyone handled the possibility of a domino spilling out of a region very well. The most successful arguments excluded the lower left and upper right cells of the checkerboard from the division and argued that even if a domino from one of the triangular regions impinged upon them, the imbalance between the white and black squares available did not allow for a complete covering of all cells. A few students looked at a case-by-case analysis according to how the lower left and upper right cells might or might not be covered, but this made for a long and detailed solution. This problem was posed by the late Robert Barrington Leigh.

Problem 2: Virtually everyone tried this question, but it proved to be difficult to pin down. One good approach was to dispose of the possibility that the triangles were isosceles (and hence congruent) to begin with, and then assume that they were scalene. For non-congruent triangles, the observation that the lengths of the longest side of the larger triangle and the shortest side of the smaller triangle could not be matched in the other triangle avoided having to consider a large number of cases and allowed for a fairly short argument. Many students who tried an exhautive case by case pairing of the sides of the two triangles missed out cases and made assumptions that were not justified. While everyone who solved the problem understood the role of the triangle inequality for the side lengths, many were careless in setting up the quadratic inequality that had to be satisfied by the ratio or in solving it. This problem is due to Chris Fisher.

Problem 3: There were more candidates than expected who found the example $x^2 + 4$, but a few gave no justification. A couple of students did not know what a polynomial was. The successful solutions considered the situations where $xy = x + y$, although there were some who failed to show adequately that this covered all possible values of the variable. Those who failed to find this argument often got to the point of showing that $f(0) \geq 0$ and $f(x) + f(-x) >= 0$, but either could not proceed further or messed up on the inequalities.

Problem 4: On the whole, this problem was better done than expected, with students realizing the value of showing that the operation was associative. However, there were two key observtions that were needed for full credit. It was necessary to show that the domain $0 < x < 1$ was preserved under * so that there was no possibility of the denominator vanishing and also to at least mention that the operation was commutative. A number of students recognized the expression for an n-fold combination of elements, although some did not successfully manage the details of the induction argument. Many students got part (b), which was worth 2 points. This problem is due to Jacob Tsimerman.

Problem 5: This was expected to be a difficult problem, so part (a) was appended so that students could pick up an easy 2 points and also, perhaps, be induced to consider an inversion in the incircle, as a route to success. The first place candidate in fact gave a very nice argument distinct from that projected by the examiners. Only one other student made significant progress, but failed to complete the solution. This problem is due to Naoki Sato.