Report of the Thirty Sixth Canadian Mathematical Olympiad 2004

CMS
SMC

Sun Life Financial

The UNIVERSITY of WINNIPEG

The Canadian Mathematical Olympiad (CMO) is an annual national mathematics competition sponsored by the Canadian Mathematical Society (CMS) and is administered by the Canadian Mathematical Olympiad Committee (CMO Committee), a sub-committee of the Mathematical Competitions Committee. The CMO was established in 1969 to provide an opportunity for students who performed well in various provincial mathematics competitions to compete at a national level. It also serves as preparation for those Canadian students competing at the International Mathematical Olympiad (IMO).

Students qualify to write the CMO by earning a sufficiently high score on the Canadian Open Mathematical Challenge (COMC). Students may also be nominated to write the CMO by a provincial coordinator.

The Society is grateful for support from the Sun Life Financial as the Major Sponsor of the 2004 Canadian Mathematical Olympiad and the other sponsors which include: the Ministry of Education of Ontario; the Ministry of Education of Quebec; Alberta Learning; the Department of Education, New Brunswick; the Department of Education, Newfoundland and Labrador; the Department of Education, the Northwest Territories; the Department of Education of Saskatchewan; the Department of Mathematics and Statistics, University of Winnipeg; the Department of Mathematics and Statistics, University of New Brunswick at Fredericton; the Centre for Education in Mathematics and Computing, University of Waterloo; the Department of Mathematics and Statistics, University of Ottawa; the Department of Mathematics, University of Toronto; the Department of Mathematics, University of Western Ontario; Nelson Thompson Learning; John Wiley and Sons Canada Ltd.; A.K. Peters and Maplesoft.

The provincial coordinators of the CMO are Peter Crippin, University of Waterloo ON; John Denton, Dawson College QC; Diane Dowling, University of Manitoba; Harvey Gerber, Simon Fraser University BC; Gareth J. Griffith, University of Saskatchewan; Jacques Labelle, Université du Québec à Montréal; Peter Minev, University of Alberta; Gordon MacDonald, University of Prince Edward Island; Roman Mureika, University of New Brunswick; Thérèse Ouellet, Université de Montréal QC; Donald Rideout, Memorial University of Newfoundland.

I offer my sincere thanks to the CMO Committee members who helped compose and/or mark the exam: Jeff Babb, University of Winnipeg; Robert Craigen, University of Manitoba; James Currie, University of Winnipeg; Robert Dawson, St. Mary's University; Chris Fisher, University of Regina; Rolland Gaudet, College Universitaire de St. Boniface; J. P. Grossman, Massachusetts Institute of Technology; Richard Hoshino, Dalhousie University; Kirill Kopotun, University of Manitoba; Ortrud Oellermann, University of Winnipeg; Felix Recio, University of Toronto; Naoki Sato, William M. Mercer; Anna Stokke, University of Winnipeg; Daryl Tingley, University of New Brunswick.

I am grateful to Michelle Davidson from University of Manitoba and Charlene Pawluk from University of Winnipeg for assistance with marking and to Suat Namli, Louisiana State University and Bruce Shawyer, Memorial University for submitting problems. I would like to thank Rolland Gaudet for the French translation of the Exam and Solutions and Matthieu Dufour, Université du Québec à Montréal for proofreading many of the French documents. I'm also grateful for the support provided by the CMS Mathematics Competitions Committee chaired by Peter Cass, University of Western Ontario. A project of this magnitude cannot run smoothly without a great deal of administrative assistance and I'm indebted to Nathalie Blanchard of the CMS Executive Office and Julie Beaver of the Mathematics and Statistics Department, University of Winnipeg for all their help. Finally, a special thank you must go out to Graham Wright, Executive Director of the CMS, who oversaw the entire organization of this year's contest and provided a great deal of support and encouragement. Without his commitment to the CMO, it would not be the success that it continues to be.

Terry Visentin, Chair
Canadian Mathematical Olympiad Committee

The 36th (2004) Canadian Mathematical Olympiad was held on Wednesday, March 31st, 2004. A total of 79 students from 51 schools in nine Canadian provinces were invited to write the paper; one student elected not to participate. The number of contestants from each province was as follows:

BC(12) AB(9) SK(1) MB(3) ON(45) QC(4) NB(1) NS(2) NF(1)

The 2004 CMO consisted of five questions. Each question was worth 7 marks for a total maximum score of m=35. The contestants' performances were grouped into four divisions as follows.

Division	Range of Scores	No. of Students
I	$24 \leq m \leq 35$	11
II	$19 \leq m < 24$	10
III	$14 \leq m < 19$	23
IV	$0 \leq m < 14$	34

FIRST PRIZE — Sun Life Financial Cup — $2000
Yufei Zhao
Don Mills Collegiate Institute

SECOND PRIZE — $1500
Jacob Tsimerman
University of Toronto Schools, Toronto, Ontario

THIRD PRIZE — $1000
Dong Uk (David) Rhee
McNally High School, Edmonton, Alberta

HONOURABLE MENTIONS — $500
Boris Braverman
Simon Fraser Junior High, Calgary, Alberta

Dennis Chuang
Strathcona-Tweedsmuir School, Okotoks, Alberta

Gabriel Gauthier-Shalom
Marianopolis College, Montreal, Quebec

Oleg Ivrii
Don Mills Collegiate Institute, Don Mills, Ontario

János Kramár
University of Toronto Schools, Toronto, Ontario

Andrew Mao
A.B. Lucas Secondary School, London, Ontario

Richard Peng
Vaughan Road Academy, Toronto, Ontario

Peng Shi
Sir John A. MacDonald Collegiate Institute, Agincourt, Ontario

Division 2

$19 \leq m < 24$

Mehdi Abdeh-Kolahchi	Halifax West High School	NS
Andrew James Critch	Clarenville Integrated High School	NL
Elyot Grant	Cameron Heights Collegiate Institute	ON
Sung Hwan Hong	Port Moody Secondary School	BC
Taotao Liu	Vincent Massey Secondary School	ON
Charles Qi	Jarvis Collegiate Institute	ON
Yehua Wei	York Mills Collegiate Institute	ON
Tom Yue	A.Y. Jackson Secondary School	ON
Peter Zhang	Sir Winston Churchill High School	AB
John Zhou	Centennial Secondary School	BC

Division 3

$14 \leq m < 19$

Rongtao Dan	Point Grey Secondary School	BC
Lin Fei	Sir Winston Churchill S.S.	BC
Steve Kim	Port Moody Secondary School	BC
Michael Lipnowski	St. John's-Ravenscourt School	MB
Tiffany Liu	A.Y. Jackson Secondary School	ON
Yang Liu	Francis Libermann Collegiate H.S.	ON
Amirali Modir Shanechi	Don Mills Collegiate Institute	ON
Chunpo Pan	Jarvis Collegiate Institute	ON
Jennifer Park	Bluevale Collegiate Institute	ON
Karol Przybytkowski	Marianopolis College	QC
Roman Shapiro	Vincent Massey Secondary School	ON
Chen Shen	A.Y. Jackson Secondary School	ON
Jimmy Shen	Vincent Massey Secondary School	ON
Geoffrey Siu	London Central Secondary School	ON
John Sun	Vincent Massey Secondary School	ON
Kuan Chieh Tseng	Yale Secondary School	BC
Shaun White	Vincent Massey Secondary School	ON
Lilla Yan	Erindale Secondary School	ON
Ti Yin	William Lyon Mackenzie C.I.	ON
Allen Zhang	Burnaby South Secondary School	BC
Ken Zhang	Western Canada High School	AB
Yin Zhao	Vincent Massey Secondary School	ON
Ivy Zou	Earl Haig Secondary School	ON

Division 4

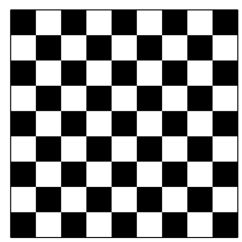
$0 \leq m < 14$

Mu Cai	Salisbury Composite High School	AB
Qi Chen	Cornwall C. I. & V. S.	ON
Francis Chung	A.B. Lucas Secondary School	ON
Bo Hong Deng	Jarvis Collegiate Institute	ON
Robert Embree	Dr. John Hugh Gillis School	NS
Matthew Folz	Port Moody Secondary School	BC
Yin Ge	Marianopolis College	QC
Will Guest	St. John's-Ravenscourt School	MB
Weibo Hao	Vincent Massey Secondary School	ON
Luke Yen Chun Hsieh	Kitsilano Secondary School	BC
Chen Huang	Sir Winston Churchill S.S.	BC
Kent Huynh	University of Toronto Schools	ON
Charley Jiang	Vincent Massey Secondary School	ON
Jaehun Kim	Bayview Secondary School	ON
Jaeseung Kim	Bayview Secondary School	ON
Koji Kobayashi	David Thompson Secondary School	BC
Sue Jean Lee	Bishop Strachan School	ON
Qing Li	St. John's Kilmarnock School	ON
Jerry Lo	Vernon Barford School	AB
Sukwon Oh	Martingrove Collegiate Institute	ON
Neeraj Sood	Westdale Secondary School	ON
Evan Stratford	University of Toronto Schools	ON
Sarah Sun	Holy Trinity Academy	AB
Frank Wang	Vincent Massey Secondary School	ON
Joyce Xie	Burnaby South Secondary School	BC
Brian Yu	Old Scona Academic High School	AB
Bo Yang Yu	Saint John High School	NB
Jiantao Yu	Columbia International College	ON
Tianyao Yu	Columbia International College	ON
Guan Zhang	Grant Park High School	MB
Si Zhang	Aden Bowman Collegiate Institute	SK
Tianxing Zhang	Vanier College	QC
Ryan Zhou	Adam Scott C. & V. I.	ON
Siqi Zhu	Earl Haig Secondary School	ON

36th Canadian Mathematical Olympiad
March 31, 2004

1. Find all ordered triples (x, y, z) of real numbers which satisfy the following system of equations:
$$\begin{cases} xy &= z - x - y \\ xz &= y - x - z \\ yz &= x - y - z \end{cases}$$

2. How many ways can 8 mutually non-attacking rooks be placed on the 9×9 chessboard (shown here) so that all 8 rooks are on squares of the same colour?
[Two rooks are said to be attacking each other if they are placed in the same row or column of the board.]



3. Let A, B, C, D be four points on a circle (occurring in clockwise order), with $AB < AD$ and $BC > CD$. Let the bisector of angle BAD meet the circle at X and the bisector of angle BCD meet the circle at Y. Consider the hexagon formed by these six points on the circle. If four of the six sides of the hexagon have equal length, prove that BD must be a diameter of the circle.

4. Let p be an odd prime. Prove that
$$\sum_{k=1}^{p-1} k^{2p-1} \equiv \frac{p(p+1)}{2} \pmod{p^2}.$$
[Note that $a \equiv b \pmod{m}$ means that $a - b$ is divisible by m.]

5. Let T be the set of all positive integer divisors of 2004^{100}. What is the largest possible number of elements that a subset S of T can have if no element of S is an integer multiple of any other element of S?

Solutions to the 2004 CMO

written March 31, 2004

1. Find all ordered triples (x, y, z) of real numbers which satisfy the following system of equations:

$$\begin{cases} xy &= z - x - y \\ xz &= y - x - z \\ yz &= x - y - z \end{cases}$$

Solution 1

Subtracting the second equation from the first gives $xy - xz = 2z - 2y$. Factoring $y - z$ from each side and rearranging gives

$$(x + 2)(y - z) = 0,$$

so either $x = -2$ or $z = y$.

If $x = -2$, the first equation becomes $-2y = z + 2 - y$, or $y + z = -2$. Substituting $x = -2$, $y + z = -2$ into the third equation gives $yz = -2 - (-2) = 0$. Hence either y or z is 0, so if $x = -2$, the only solutions are $(-2, 0, -2)$ and $(-2, -2, 0)$.

If $z = y$ the first equation becomes $xy = -x$, or $x(y + 1) = 0$. If $x = 0$ and $z = y$, the third equation becomes $y^2 = -2y$ which gives $y = 0$ or $y = -2$. If $y = -1$ and $z = y = -1$, the third equation gives $x = -1$. So if $y = z$, the only solutions are $(0, 0, 0)$, $(0, -2, -2)$ and $(-1, -1, -1)$.

In summary, there are 5 solutions: $(-2, 0, -2)$, $(-2, -2, 0)$, $(0, 0, 0)$, $(0, -2, -2)$ and $(-1, -1, -1)$.

Solution 2

Adding x to both sides of the first equation gives

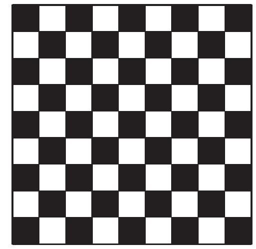
$$x(y + 1) = z - y = (z + 1) - (y + 1) \ \Rightarrow\ (x + 1)(y + 1) = z + 1.$$

Similarly manipulating the other two equations and letting $a = x + 1$, $b = y + 1$, $c = z + 1$, we can write the system in the following way.

$$\begin{cases} ab &= c \\ ac &= b \\ bc &= a \end{cases}$$

If any one of a, b, c is 0, then it's clear that all three are 0. So $(a, b, c) = (0, 0, 0)$ is one solution and now suppose that a, b, c are all nonzero. Substituting $c = ab$ into the second and third equations gives $a^2 b = b$ and $b^2 a = a$, respectively. Hence $a^2 = 1$, $b^2 = 1$ (since a, b nonzero). This gives 4 more solutions: $(a, b, c) = (1, 1, 1)$, $(1, -1, -1)$, $(-1, 1, -1)$ or $(-1, -1, 1)$. Reexpressing in terms of x, y, z, we obtain the 5 ordered triples listed in Solution 1.

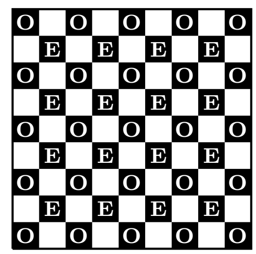
2. How many ways can 8 mutually non-attacking rooks be placed on the 9×9 chessboard (shown here) so that all 8 rooks are on squares of the same colour?
[Two rooks are said to be attacking each other if they are placed in the same row or column of the board.]



Solution 1

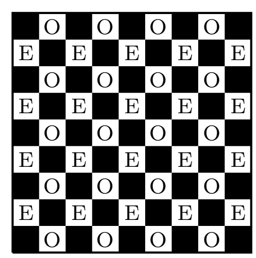
We will first count the number of ways of placing 8 mutually non-attacking rooks on black squares and then count the number of ways of placing them on white squares. Suppose that the rows of the board have been numbered 1 to 9 from top to bottom.

First notice that a rook placed on a black square in an odd numbered row cannot attack a rook on a black square in an even numbered row. This effectively partitions the black squares into a 5×5 board and a 4×4 board (squares labelled O and E respectively, in the diagram at right) and rooks can be placed independently on these two boards. There are 5! ways to place 5 non-attacking rooks on the squares labelled O and 4! ways to place 4 non-attacking rooks on the squares labelled E.



This gives 5!4! ways to place 9 mutually non-attacking rooks on black squares and removing any one of these 9 rooks gives one of the desired configurations. Thus there are $9 \cdot 5!4!$ ways to place 8 mutually non-attacking rooks on black squares.

Using very similar reasoning we can partition the white squares as shown in the diagram at right. The white squares are partitioned into two 5×4 boards such that no rook on a square marked O can attack a rook on a square mark E. At most 4 non-attacking rooks can be placed on a 5×4 board and they can be placed in $5 \cdot 4 \cdot 3 \cdot 2 = 5!$ ways. Thus there are $(5!)^2$ ways to place 8 mutually non-attacking rooks on white squares.



In total there are $9 \cdot 5!4! + (5!)^2 = (9+5)5!4! = 14 \cdot 5!4! = 40320$ ways to place 8 mutually non-attacking rooks on squares of the same colour.

Solution 2

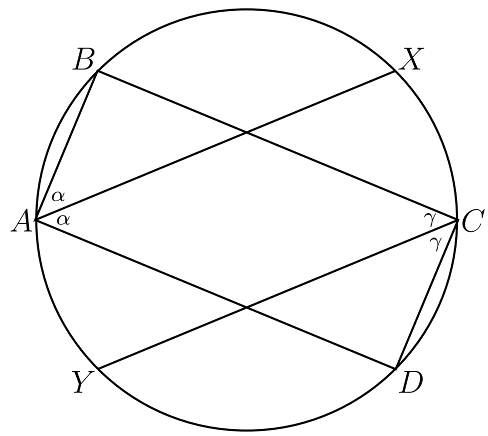
Consider rooks on black squares first. We have 8 rooks and 9 rows, so exactly one row will be without rooks. There are two cases: either the empty row has 5 black squares or it has 4 black squares. By permutation these rows can be made either last or second last. In each case we'll count the possible number of ways of placing the rooks on the board as we proceed row by row.

In the first case we have 5 choices for the empty row, then we can place a rook on any of the black squares in row 1 (5 possibilities) and any of the black squares in row 2 (4 possibilities). When we attempt to place a rook in row 3, we must avoid the column containing the rook that was placed in row 1, so we have 4 possibilities. Using similar reasoning, we can place the rook on any of 3 possible black squares in row 4, etc. The total number of possibilities for the first case is $5 \cdot 5 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 = (5!)^2$. In the second case, we have 4 choices for the empty row (but assume it's the second last row). We now place rooks as before and using similar logic, we get that the total number of possibilities for the second case is $4 \cdot 5 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 4(5!4!)$.

Now, do the same for the white squares. If a row with 4 white squares is empty (5 ways to choose it), then the total number of possibilities is $(5!)^2$. It's impossible to have a row with 5 white squares empty, so the total number of ways to place rooks is

$$(5!)^2 + 4(5!4!) + (5!)^2 = (5 + 4 + 5)5!4! = 14(5!4!).$$

3. Let A, B, C, D be four points on a circle (occurring in clockwise order), with $AB < AD$ and $BC > CD$. Let the bisector of angle BAD meet the circle at X and the bisector of angle BCD meet the circle at Y. Consider the hexagon formed by these six points on the circle. If four of the six sides of the hexagon have equal length, prove thatBD must be a diameter of the circle.



Solution 1

We're given that $AB < AD$. Since CY bisects $\measuredangle BCD$, $BY = YD$, so Y lies between D and A on the circle, as in the diagram above, and $DY > YA$, $DY > AB$. Similar reasoning confirms that X lies between B and C and $BX > XC$, $BX > CD$. So if $ABXCDY$ has 4 equal sides, then it must be that $YA = AB = XC = CD$.

Let $\measuredangle BAX = \measuredangle DAX = \alpha$ and let $\measuredangle BCY = \measuredangle DCY = \gamma$. Since $ABCD$ is cyclic, $\measuredangle A + \measuredangle C = 180°$, which implies that $\alpha + \gamma = 90°$. The fact that $YA = AB = XC = CD$ means that the arc from Y to B (which is subtended by $\measuredangle YCB$) is equal to the arc from X to D (which is subtended by $\measuredangle XAD$). Hence $\measuredangle YCB = \measuredangle XAD$, so $\alpha = \gamma = 45°$. Finally, BD is subtended by $\measuredangle BAD = 2\alpha = 90°$. Therefore BD is a diameter of the circle.

Solution 2

We're given that $AB < AD$. Since CY bisects $\measuredangle BCD$, $BY = YD$, so Y lies between D and A on the circle, as in the diagram above, and $DY > YA$, $DY > AB$. Similar reasoning confirms that X lies between B and C and $BX > XC$, $BX > CD$. So if $ABXCDY$ has 4 equal sides, then it must be that $YA = AB = XC = CD$. This implies that the arc from Y to B is equal to the arc from X to D and hence that $YB = XD$. Since $\measuredangle BAX = \measuredangle XAD$, $BX = XD$ and since $\measuredangle DCY = \measuredangle YCB$, $DY = YB$. Therefore $BXDY$ is a square and its diagonal, BD, must be a diameter of the circle.

4. Let p be an odd prime. Prove that

$$\sum_{k=1}^{p-1} k^{2p-1} \equiv \frac{p(p+1)}{2} \pmod{p^2}.$$

[Note that $a \equiv b \pmod{m}$ means that $a-b$ is divisible by m.]

Solution

Since $p-1$ is even, we can pair up the terms in the summation in the following way (first term with last, 2nd term with 2nd last, etc.):

$$\sum_{k=1}^{p-1} k^{2p-1} = \sum_{k=1}^{\frac{p-1}{2}} \left(k^{2p-1} + (p-k)^{2p-1}\right).$$

Expanding $(p-k)^{2p-1}$ with the binomial theorem, we get

$$(p-k)^{2p-1} = p^{2p-1} - \cdots - \binom{2p-1}{2} p^2 k^{2p-3} + \binom{2p-1}{1} pk^{2p-2} - k^{2p-1},$$

where every term on the right-hand side is divisible by p^2 except the last two. Therefore

$$k^{2p-1} + (p-k)^{2p-1} \equiv k^{2p-1} + \binom{2p-1}{1} pk^{2p-2} - k^{2p-1} \equiv (2p-1)pk^{2p-2} \pmod{p^2}.$$

For $1 \le k < p$, k is not divisible by p, so $k^{p-1} \equiv 1 \pmod{p}$, by Fermat's Little Theorem. So $(2p-1)k^{2p-2} \equiv (2p-1)(1^2) \equiv -1 \pmod{p}$, say $(2p-1)k^{2p-2} = mp - 1$ for some integer m. Then

$$(2p-1)pk^{2p-2} = mp^2 - p \equiv -p \pmod{p^2}.$$

Finally,

$$\begin{aligned} \sum_{k=1}^{p-1} k^{2p-1} &\equiv \sum_{k=1}^{\frac{p-1}{2}} (-p) \equiv \left(\frac{p-1}{2}\right)(-p) \pmod{p^2} \\ &\equiv \frac{p-p^2}{2} + p^2 \equiv \frac{p(p+1)}{2} \pmod{p^2}. \end{aligned}$$

5. Let T be the set of all positive integer divisors of 2004^{100}. What is the largest possible number of elements that a subset S of T can have if no element of S is an integer multiple of any other element of S?

Solution

Assume throughout that a, b, c are nonnegative integers. Since the prime factorization of 2004 is $2004 = 2^2 \cdot 3 \cdot 167$,

$$T = \left\{ 2^a 3^b 167^c \,\middle|\, 0 \leq a \leq 200,\ 0 \leq b, c \leq 100 \right\}.$$

Let

$$S = \left\{ 2^{200-b-c} 3^b 167^c \,\middle|\, 0 \leq b, c \leq 100 \right\}.$$

For any $0 \leq b, c \leq 100$, we have $0 \leq 200-b-c \leq 200$, so S is a subset of T. Since there are 101 possible values for b and 101 possible values for c, S contains 101^2 elements. We will show that no element of S is a multiple of another and that no larger subset of T satisfies this condition.

Suppose $2^{200-b-c}3^b167^c$ is an integer multiple of $2^{200-j-k}3^j167^k$. Then

$$200 - b - c \geq 200 - j - k, \quad b \geq j, \quad c \geq k.$$

But this first inequality implies $b + c \leq j + k$, which together with $b \geq j$, $c \geq k$ gives $b = j$ and $c = k$. Hence no element of S is an integer multiple of another element of S.

Let U be a subset of T with more than 101^2 elements. Since there are only 101^2 distinct pairs (b, c) with $0 \leq b, c \leq 100$, then (by the pigeonhole principle) U must contain two elements $u_1 = 2^{a_1}3^{b_1}167^{c_1}$ and $u_2 = 2^{a_2}3^{b_2}167^{c_2}$, with $b_1 = b_2$ and $c_1 = c_2$, but $a_1 \neq a_2$. If $a_1 > a_2$, then u_1 is a multiple of u_2 and if $a_1 < a_2$, then u_2 is a multiple of u_1. Hence U does not satisfy the desired condition.

Therefore the largest possible number of elements that such a subset of T can have is $101^2 = 10201$.

GRADER'S REPORT

Each question was worth a maximum of 7 marks. Every solution on every paper was graded by two different markers. If the two marks differed by more than one point, the solution was reconsidered until the difference was resolved. If the two marks differed by one point, the average was used in computing the total score. The top papers were then reconsidered until the committee was confident that the prize-winning contestants were ranked correctly.

The various marks assigned to each solution are displayed below, as a percentage. As described above, fractional scores are possible, but for the purpose of this table, marks are rounded up. So, for example, 59.0% of the students obtained a score of 6.5 or 7 on the first problem. This indicates that on 59% of the papers, at least one marker must have awarded a 7 on question #1.

Marks	**#1**	**#2**	**#3**	**#4**	**#5**
0	2.6	12.8	15.4	56.4	33.3
1	10.3	15.4	21.8	10.3	34.6
2	11.5	11.5	9.0	10.3	15.4
3	5.1	10.3	10.3	1.3	2.6
4	2.6	6.4	5.1	1.3	5.1
5	0.0	1.3	6.4	2.6	2.6
6	9.0	10.3	19.2	2.6	1.3
7	59.0	32.1	12.8	15.4	5.1

At the outset our marking philosophy was as follows: A score of 7 was given for a completely correct solution. A score of 6 indicated a solution which was essentially correct, but with a very minor error or omission. Very significant progress had to be made to obtain a score of 3. Even scores of 1 or 2 were not awarded unless some significant work was done. Scores of 4 and 5 were reserved for special situations. This approach seem to work quite well for all of the problems on this paper.

PROBLEM 1

This problem was very well done. There are many ways to manipulate the equations in this system, but no one presented a solution which differed substantially in character from the two official solutions. Students who found some, but not all, of the triples usually received one or two marks, depending on the quality of the reasoning given. One common error which caused students to miss some solutions was to cancel a factor from both sides of an equation without considering when that factor might be zero. Students who found all the triples plus some extraneous ones usually obtained three marks. Typically this occurred when students squared equations or multiplied some together.

PROBLEM 2

This problem was fairly well done with most students taking an approach similar to Solution 2 of the official solutions. Every correct solution broke the problem up into two cases: rooks on black squares vs. rooks on white squares. Students who solved just one of the two cases correctly (usually the black case) received three marks. Some students failed to discern precisely how the two cases differ. Another common error was to not take into account the number of choices for an empty row or column.

PROBLEM 3

This geometry problem was solved by quite a few students. All of the correct solutions used classical geometry with the two official solutions being the most common approaches. The major difficulty for most was to clearly explain that the 4 equal sides of the hexagon must be YA, AB, XC and CD. Quite a few long confusing arguments were presented and several other students considered many different cases, some of which were not necessary. However, once this was fact was established, no one had any trouble showing that BD is a diameter. Students who showed that XY is a diameter (which only depends on the definition of X and Y, not on any other properties of the hexagon) received 3 marks.

PROBLEM 4

Twelve students obtained full marks for this problem and all of the correct solutions were very similar to the official one. Most contestants either knew exactly how to solve it or didn't make very much progress at all. The key idea needed is to pair up the kth term of the series with the $(p-k)$th term. If this observation was made, students usually knew how to proceed from here.

PROBLEM 5

Few students made significant progress on this challenging problem. The five students who attained a mark of 6 or 7 all used the same approach as the one used in the official solutions. Students who factored 2004^{100} and gave a concise description of the set T received one mark. Students who found the maximal set S, but couldn't prove that a larger set didn't exist were awarded 3 marks.