ABSTRACTS / RÉSUMÉS DE CONFÉRENCES

COXETER-JAMES LECTURE / CONFÉRENCE COXETER-JAMES

MACIEJ ZWORSKI, University of California at Berkeley, Berkeley, California, USA
The inverse problem for resources

DOCTORAL PRIZE LECTURE / CONFÉRENCE DU PRIX DOCTORAL

JIAN SHEN, Queen's University, Kingston, Ontario
On the Caccetta-Haggkvist conjecture and some related conjectures

ADRIAN POULIOT LECTURE / CONFÉRENCE

PLENARY SPEAKERS / CONFÉRENCIERS PRINCIPAUX

ANDREAS DRESS, University of Bielefeld, Germany
Virtual crystallography and tiling theory

ANDRIANO GARSIA, University of California at San Diego, San Diego, California, USA
Update on the n! conjecture

FRANCOIS LALONDE,
Symplectic topology in dimensions 2 *and* 4

DAVID LAY, Department of Mathematics, University of Maryland, College Park, Maryland 20742-4015, USA
Recent advances in teaching linear algebra

In recent years, the exponential increase in computing power has driven a corresponding increase in the use of linear algebraic techniques to model and solve engineering and scientific problems that were computationally inaccessible a few decades ago. The language of linear algebra has become an essential part of the basic training of engineers and computer scientists, and this has influenced both the content and modes of instruction of linear algebra courses. The lecture will focus on three areas in which significant advances have been made—mathematical content, pedagogy, and the use of technology.

(1) The mathematical focus of a first course in linear algebra has shifted somewhat, from an abstract inward-looking subject to a more concrete matrix-oriented approach. Yet the insistence on precision of language and careful proof remains.

(2) Advances in the mathematical presentation of linear algebra have simplified the teaching of the material and made it more accessible to students. Improved pedagogy has also resulted from

the influence of reform in the calculus courses and from a new understanding of how mathematical concepts are internalized by average students.

(3) Technology is, of course, widely used in linear algebra courses. But its use is driven more by the needs of the students to understand the its role in linear algebra than by the popularity of technology for collegiate instruction. The lecture will conclude with a discussion of how technology is being used today in linear algebra instruction.

ELLIOT H. LIEB, Princeton University
The Quantum-mechanical world view: a remarkably successful but still incomplete theory

This talk will be about some important, unsolved problems of a mathematical nature connected with the quantum mechanical many body theory. It will center on the stability of matter problem and how this is connected to the largely unsolved problem of quantum electrodynamics. The historical background and the present status of the subject will be reviewed.

PAVEL PEVZNER, Department of Mathematics, Computer Science and Molecular Biology, University of Southern California, California, USA
Transforming mice into men

Many people (including myself) believe that transformationsof humans into mice happen only in fairy tales. However, despite some differences in appearance and habits, men and mice are genetically very similar. In the pioneering paper, Nadeau and Taylor, 1984 estimated that surprisingly few genomic rearrangements (approximately 180) happened since the divergence of human and mouse 80 million years ago. However, their analysis is non-constructive and no rearrangement scenario for human-mouse evolution has been suggested yet. The problem is complicated by the fact that rearrangements in multi-chromosomal genomes include inversions, translocations, fusions and fissions of chromosomes, a rather complex set of operations. As a result, at the first glance, a polynomial algorithm for the genomic distance problem with all these operations looks almost as improbable as the transformation of a (real) man into a (real) mouse. We prove a duality theorem which expresses the genomic distance in terms of easily computable parameters reflecting different combinatorial properties of sets of strings. This theorem leads to a polynomial-time algorithm for computing most parsimonious rearrangement scenarios. Based on this result and the latest comparative physical mapping data we have constructed a scenario of human-mouse evolution with 131 reversals/translocations/fusions/fissions. A combination of the genome rearrangement algorithm with new experimental techniques for deriving gene orders suggests a new approach to the 100-year old problem of reconstructing mammalian evolution.

This is a joint work with Sridhar Hannenhalli.

ZHIHONG XIA, Northwestern and Georgia Tech
Hamiltonian dynamical systems

We will discuss some of the recent progresses in Hamiltonian Systems and Symplectic Diffeomorphisms, particularly on the problems concerning stability and instability, Arnold diffusion in high dimensional systems.

PUBLIC LECTURE / CONFÉRENCE PUBLIQUE

JENNIFER CHAYES, Microsoft Research, Redmond, Washington, USA
Phase transitions in computer science: what makes hard problems hard

What makes hard problems hard? Understanding intractability is one of the fundamental problems in computer science. Even in the context of specific algorithms, it is often not obvious why certain

problems are difficult. In this talk, I will discuss how concepts from physics, especially from the theory of phase transitions, allow us to understand the hardness of specific random algorithms, in particular Monte Carlo algorithms for certain magnetic models. I will also show how an insight into the phase structure of combinatorial models allows us to improve these algorithms. Finally, this analysis suggests why it may be so difficult to find substantially improved algorithms for certain problems in disordered systems. This is a new, cross-disciplinary approach to the study of algorithms and intractability. No prior knowledge of phase transitions, magnetic models, algorithms or intractability is assumed in this talk.

Algebraic Geometric Methods in Differential Equations: The 20-th century in celestial mechanics and one century of work on Hilbert's 16-th problem / Méthodes algébriques et géométriques en équations différentielles: la mécanique céleste au 20è siècle et un siècle de travail sur le 16è problème de Hilbert

(A. Mingarelli and C. Rousseau, Organizers)

JACQUES BELAIR, Université de Montréal, Département de mathématiques et de statistique, Montréal, Québec H3C 3J7

Tores invariants et bistabilité de solutions périodiques dans un système d'équations différentielles à retards

The construction of normal forms has been shown to be applicable to systems of delay-differential equations to analyse degenerate (codimension higher than one) bifurcations of stationary solutions. In most instances, this investigation can only be performed using a computer-assisted approach.

We present a model of the insulin-glucose regulatory system for which this technique has been recently applied to show the presence of complicated oscillations, in the form of invariant tori and simultaneous existence of two stable periodic solutions. This system takes the form of two differential equations containing two time delays. The different feedback interactions between the components are quantitatively estimated using data from the clinical literature.

Joint work with Vincent Lemaire; supported by NSERC and FCAR.

SUE ANN CAMPBELL, University of Waterloo, Waterloo, Ontario N2L 3G1

Calculating center manifolds for delay equations in maple

We present an algorithm for symbolically computing centre manifolds for delay-differential equations when an arbitrary (finite) number of eigenvalues of a characteristic equation have zero real part. The implementation of this algorithm in the symobolic algebra language Maple is discussed, and examples of its application to the investigation of various bifurcations are presented.

FLORIN DIACU, Department of Mathematics and Statistics, University of Victoria, Victoria, British Columbia V8W 3P4

Dynamical systems given by quasihomogeneous potentials

Quasihomogeneous functions are given by sums of homogeneous functions. They have wide applications in physics (Liboff and Coulomb potentials), astronomy (Newton and Manev potentials), and chemistry (Van der Waals and Lennrad-Jones potentials. The goal of this talk is to present some qualitative properties of dynamical systems given by quasihomogeneous potentials.

FREDDY DUMORTIER, Limburgs Universitair Centrum, Diepenbeek, Belgium

Limit cycles in Liénard equations

The talk will deal with (generalized) polynomial Liénard equations, *i.e.* 2-dimensional differential equations coming from second order scalar equations $\frac{d^2x}{dt^2} + P(x)\frac{dx}{dt} + Q(x) = 0$, with P and Q polynomials. Depending on the degree of P and Q there are results concerning the number of limit cycles which such systems can have in the phase plane. We will present some of these results and discuss some open problems.

JEAN PIERRE FRANCOISE, Université P. M. Curie, Paris VI, France
The local Hilbert's 16th problem and its generalization to any dimension

Several new techniques have been recently developped in the framework of solving the local Hilbert's 16-th problem for 2-dimensional systems. The algorithm of the successives derivatives, algebraic geometry approach based on Bautin's ideal and projection of analytic varieties will be shortly displayed. A generalization to any dimension of these techniques will be discussed.

JOHN GUCKENHEIMER, Cornell University, Ithaca, New York, New York, USA
Computing periodic orbits of vector fields

Periodic orbits are fundamental structures of flows. New methods for computing periodic orbits and their bifurcations with high accuracy are being developed with the use of automatic differentiation. This lecture will present these methods, make comparisons with other methods and discuss outstanding problems. Case studies include stiff systems with multiple time scales and the computation of canards.

ANA GUZMAN, Mexico
To be announced

PHIL HOLMES, Department of Mechanical and Aerospace Engineering and Program in Applied and Computational Mathematics, Princeton University, Princeton, New Jersey 08544-1000, USA
Non-holonomic and piecewise-holonomic mechanical systems

Nonholonomic (velocity dependent) constraints can lead to asymptotically stable motions in certain conservative mechanical systems; the Chaplygin sleigh is a canonical example. In studying models for legged locomotion, piecewise-holonomic constraints (due to intermittent foot placements) are typical. The resulting hybrid dynamical systems include flows along a smooth vectorfield punctuated by impulsive jumps governed by discrete 'collision maps.' They may be viewed as generalisations of billiards-type problems. Such systems can also exhibit partial asymptotic stability, even while conserving total energy. I will describe joint work with Michael Coleman (Cornell University) and John Schmitt (Princeton University) on a discrete sister to the Chaplygin sleigh, and on a simple model for rapidly running insects, which illustrate this phenomenon.

YULIK IL'YASHENKO, Cornell University and oscow University
Restricted versions of Hilbert 16th problem and related topics

Survey of different versions of Hilbert 16th problem is presented. Some new simplifications are proposed. Number of limit cycles of Abel equation with bounded coefficients is estimated from above. Picard-Fuchs equation for Abelian integrals is written.

KENNETH MEYER, University of Cincinnati, Department of Mathematics, Cincinnati Ohio 45221
Integral manifolds of the restricted three-body problem

We compute the homology of the integral manifolds of the restricted three-body problem—planar and spatial, unregularized and regularized. Holding the Jacobi constant fixed defines a three dimensional algebraic set in the planar case and a five dimensional algebraic set in the spatial case (the integral manifolds). The singularities of the restricted problem due to collusions are removable which defines the regularized problem.

There are five positive critical values of the Jacobi constant: one is due to a critical point at infinity, another is due to the Lagrangian critical points, and three are due to the Eulerian critical points. The critical point at infinity occurs only in the spatial problems. We compute the homology of the integral manifold for each regular value of the Jacobi constant. These computations show that at each critical value the integral manifolds undergo a bifurcation in their topology. The bifurcation due to a critical point at infinity shows that Birkhoff's conjecture is false even in the restricted problem.

Birkhoff also asked if the planar problem is the boundary of a cross section for the spatial problem. Our computations and homological criteria show that the answer is no. Also, we discuss the existence of global cross sections.

R. MOECHEL, Minnesota
To be announced

DAN OFFIN,
Variational structure of the domains of stability

The classical approach to studying parametric resonance in conservative scalar second order differential equations uses Floquet theory. In modern textbooks this is used to prove the celebrated result of Liapunov which states that strong stability corresponds exactly to the condition $|\text{trace}\, R(\pi)| < 2$. The transition from stability to instability then occurs when $|\text{trace}\, R(\pi)| = 2$ where $R(t)$ denotes the fundamental matrix solution normalized so that $R(0) = I$. In this talk we present a different approach, using symplectic geometry. We obtain a completely equivalent criterion to Liapunov's, in terms of the indices of certain variational problems. These indices are interpreted as the rotation and intersection number of certain Lagrangian planes. The main theorem applies to planar quadratic Hamiltonian systems which are periodic in the time variable and which satisfy the classical condition of Legendre. This result has applications to nonlinear problems, as well as special results assuming symmetry of the coefficient matrix, in the case of higher dimensional systems.

ERNESTO PEREZ-CHAVELA, UAM, Mexico
Heteroclinic phenomena in the Sitnikov problem

We give the deduction of a Melnikov function for the Sitnikov problem. Using a perturbation method introduced by Melnikov and a thorough analysis of the geometry of certain auxiliary functions that we introduce, we prove analytically the existence of transverse heteroclinic orbits. As a consequence, we can embed a Bernoulli shift near these orbits, showing that the Sitnikov problem possesses chaotic dynamics.

ROBERT ROUSSARIE, Université de Bourgogne, France
Melnikov functions and Bautin ideal

The computation of the number of limit cycles which appear in an analytic unfolding of planar vector fields is related to the decomposition of the displacement function of this unfolding in an ideal of functions in the parameter space, called the Ideal of Bautin. On the other hand, the asymptotic of the displacement function, for 1-parameter unfoldings of hamiltonian vector fields is given by Melnikov functions which are defined as the coefficients of Taylor expansion in the parameter. It is

interesting to compare these two notions and to study if the general estimations of the number of limit cycles in terms of the Bautin ideal could be reduced to the computations of Melnikov functions for some 1-parameter subfamilies.

CHRISTIANE ROUSSEAU, Université de Montréal, Montréal, Québec
Finiteness part of Hilbert's 16th problem for quadratic vector fields

In 1991 Dumortier, Roussarie and Rousseau presented a program to prove the finiteness part of Hilbert's 16th problem for quadratic vector fields, namely the existence of a uniform bound for the number of limit cycles of a quadratic vector field. The program reduced the proof to 121 local problems consisting in proving the finite cyclicity of 121 graphics arising in quadratic systems. We present the recent progress in this program, what is done and what difficulties still remain.

D. SAARI, Northwestern
To be announced

DANA SCHLOMIUK, Université de Montréal, Montréal, Québec
Hilbert's 16th problem—meeting ground of analysis, algebra, algebra and geometry

After briefly rviewing some successes and failures of the past ten years, we discuss some recent work and look at challenges ahead.

TADASHI TOKIEDA, UQAM
Perturbation theory for symmetric hamiltonian systems

The problem of persistence, bifurcation, stability of periodic orbits and equilibria is of special importance in perturbation theory, and many results are known (*e.g.* Poincaré, Weinstein, Moser). These results generalize to systems that have symmetries given by a hamiltonian action of a Lie group. The natural objects of study are *relative periodic orbits* (orbits that are closed up to group action) and *relative equilibria*. We derive a recipe for reducing the theory to the classical theory, which works even in the hard case when we reduce at a singular value of the moment map (as we often must in real life). Plane vortices are discussed as examples; moreover, some of their exact solutions show that the hypothesis in our recipe is tight. [joint work with Lerman and Montaldi]

SERGEY YAKOVENKO, Weizmann Institute, Israël
Tangential Hilbert's 16-th problem and differential equations

We discuss some recent achievements related to bifurcations of limit cycles from periodic orbits of planar Hamiltonian systems. In a somewhat inexpected way, all progress in counting zeros of Abelian integrals (the qualitative property of a completely algebraic object) was achieved by applying purely transcendental methods of "quantitative" theory of ordinary differential equations.

Applied Logic / Logique appliquée

(W. MacCaull, P. Panangaden and P. Scott, Organizers)

FAHIEM BACCHUS, Department of Computer Science, University of Toronto, Toronto Ontario
A search engine based on model checking

Search and declarative representations are two of the most important themes in AI research. Many problems in AI (and Computer Science in general) are most effectively solved by search, and declarative representations of the knowledge required to specify and solve these problems offer many advantages. In this talk I will show how a general purpose search engine can be constructed from the simple idea of evaluating logical formulas against finite, or more generally, recursively enumerable models. The system can then be configured to solve particular search problems by simply specifying those problems using appropriate collections of logical formulas. Advice as to how to solve the problem can also be supplied by including temporal logic formulas that are used to help guide the search. The resulting system has much in common with the "programming in logic" paradigm advanced by Prolog, but offers the user much more flexible control over the underlying search procedure.

RICK BLUTE, Department of Mathematics and Statistics, University of Ottawa, Ottawa, Ontario K1N 6N5
Nuclear ideals

Closed categories, either monoidal or cartesian, provide the foundation for the categorical modelling of logics, such as linear or intuitionistic logic. In this work, I will introduce a new categorical construction, the *nuclear ideal*. These exist within monoidal categories in which only certain of the morphisms allow the sort of transpositions implied by the existence of a closed structure. Examples arise in the category of Hilbert spaces, and a category of distributions on Euclidean space.

This work can also be viewed as a generalization of the category of relations. Indeed the original motivation for this study came from attempts to generalize the structure of the category of relations to handle what might be called "probabilistic relations" with an eye towards certain applications in computer science.

We also extend the recent work of Joyal, Street and Verity on *traced monoidal categories* to this setting by introducing the notion of a *trace ideal*. Thus our work can be viewed as modelling a generalized form of Girard's Geometry of Interaction, an extremely novel approach to the semantics of proof theory designed to capture the dynamics of normalization.

Finally, we will mention some of the possible applications of nuclear ideals to the categorical structures arising from topological quantum field theory and conformal field theory.

This talk will summarize joint work with S. Abramsky, P. Panangaden and D. Pronk.

MARTA BUNGE, Department of Mathematics and Statistics, University of McGill Montréal, Québec, H3A 2K6
Relative stone duality

Let $e\colon \mathcal{E} \to \mathcal{S}$ be a bounded geometric morphism between elementary toposes. We prove a relative pure/entire factorization of any geometric morphism over $\mathcal{S}$ whose domain is a *dominance* (subopen and such that $\mathcal{S}$-definable monos in it compose). Closely related to it is a relative Stone Duality. Denote by $\mathrm{DL}_{\Omega_{\mathcal{S}}}(\mathcal{E})$ the category of $\Omega_{\mathcal{S}}$-distributive lattices in $\mathcal{E}$ and $\Omega_{\mathcal{S}}$-action preserving lattice homomorphisms, and by $\mathrm{FRM}_{\Omega_{\mathcal{S}}}(\mathcal{E})$ the category of frames A in $\mathcal{E}$ for which the corresponding topos $\mathcal{E}[A]$ of sheaves on A is a dominance over $\mathcal{S}$ and frame homomorphisms. We prove that there is a duality between these categories and that it restricts to an equivalence between suitably defined categories $\mathrm{Boole}_{\Omega_{\mathcal{S}}}(\mathcal{E})$ and $\mathrm{Stone}_{\Omega_{\mathcal{S}}}(\mathcal{E})^{op}$. When $\mathcal{S}$ is *Sets*, this reduces to the usual Stone Duality. As an application, we answer a question of P. T. Johnstone (*Cartesian monads on toposes*, J. Pure Appl. Alg. **116**(1997), 199–220). This is part of ongoing work on "Distribution Algebras", joint with J. Funk (UBC), M. Jibladze (Louvain-la-Neuve) and T. Streicher (Darmstadt).)

PETER CAINES, Department of Electrical and Computer Engineering, McGill University, Montréal, Québec H3A 2A7
A COCOLOG *logic for systems and control theory*

The COCOLOG (Conditional Observer and Controller Logic) system [Caines and Wang, SIAM J. Cont Opt., 1995] is a framework for the design and implementation in first order logic of controllers for finite deterministic input-state-output machines. A COCOLOGcontrol system consists of a partially ordered family of first order logical theories expressed in the typed first order languages $\{L_k\ ;\ k \geq 0\}$ describing and enabling the controlled evolution of the state of a given partially observered finite machine $\mathcal{M}$. The machine theories, $\{\mathrm{Th}(o_1^k)\ ;\ k \geq 1\}$, depend upon the observed input-output trajectories, where new data constitutes *new axioms* $\mathrm{AXM}^{\mathrm{obs}}(L_k), k \geq 1$. The combination of these new axioms with the previous COCOLOG axioms generates a set of axioms for the new theory, denoted Σ_k. Next, the extra-logical conditional control rules $\mathrm{CCR}(L_k)$ gives (mutually exclusive and exhaustive) formulas each of which triggers a distinct control action. So as to increase the efficiency of COCOLOG, a class of restricted versions is introduced called (the systems of) *Markovian fragments* [Wei and Caines, SIAM J. Cont Opt., 1996]. Further, since the expression of relatively simple control specifications may become extremely complicated, a theory and methodology has been introduced (see [Martínez-Mascarúa and Caines, *Proc. WODES'96*) for the introduction of new together with their definitional formulas. The resulting *Macro (*COCOLOG*) Languages* are integrated with the system of so-called Macro (COCOLOG) Actions. Finally, the *dynamical consistency* notion of state aggregation for hierarchical control [Caines and Wei, *Systems and Control Letters*, 1995] permits the construction of hierarchically layered COCOLOG controllers.

ROBIN COCKETT, Department of Computer Science, University of Calgary, Calgary, Alberta, T2N 1N4

Double glueing

The glueing construction in the form of the Freyd covering is a well-known and useful tool in categorical logic. Recently Hyland and Tan showed how a variation of this construction, which they called the double glueing construction, could be used both to produce new models of linear logic and to establish the full completeness of certain existing models. These ideas were further generalized by Masahito Hasegewa in order to establish the full completeness of the translations between certain linear type theories.

The talk will discuss the generalization of these ideas to linearly distributive categories and indicate the connection between the double glueing construction and the Chu construction. In particular, a variation of the standard double glueing construction will be presented (which might be called Chu-glue). This latter construction, I claim, is the more natural construction when morphisms are understood to be linear functors.

JOSEE DESHARNAIS, School of Computer Science, University of McGill, Montreal, Quebec H3A 2A7

A logical characterization of bisimulation for labelled Markov processese

Logic formulas are used in the area of formal methods for verification to specify properties of systems and to verify equivalences and preorder relations between processes. We propose a model for interacing processes having a continuous state space. This model is probabilistic in the sense that a process reacts to a given action taken by its environment following a probability distribution on the state space. We define a logic with a very simple syntax that can characterize our notion of equivalence between processes as well as our preorder relation.

AMY FELTY, Bell Laboratories, Lucent Technologies, Murray Hill, New Jersey 07974, USA

A semantic model of types for proof-carrying code

Proof-carrying code (PCC) provides a mechanism for insuring that a host, or code consumer, can safely run code delivered by a code producer. In our PCC system, the host specifies a safety policy

as a set of inference rules in higher-order logic. In addition to a compiled program, the code producer delivers a formal proof of safety expressed in terms of those inference rules that can be easily checked. One advantage of the PCC approach to code safety is that the trusted computing base is extremely small; it includes only the proof checker for verifying the proof of safety.

Information about a particular programming language or compiler, such as the type system, is essential in constructing proofs of safety. In previous work on PCC, this information was included as new inference rules added to the base logic. Consequently, a different type checker had to be implemented for each programming language, and even for each compiler. We present a universal type framework in which we model types via definitions from first principles and then prove the typing rules as lemmas. All definitions and lemmas used in a particular proof are included inside it, and thus no modification to the proof checker is required to check it. As a result, a code consumer can use the same checker to check proofs of safety for programs compiled from different source languages. We show how to model traversal, allocation, and initialization of values in a wide variety of types, including functions, records, unions, existential types, and covariant recursive types.

ESFANDIAR HAGHVERDI, Department of Mathematics, University of Ottawa, Ottawa, Ontario K1N 6N5

Linear logic, geometry of proofs and full completeness

Girard invented Linear Logic, a resource sensitive logic which is a refinement of classical and intuitionistic logic. He introduced many novelties including a graphical proof syntax and a wide range of mathematical models. He also introduced his Geometry of Interaction Programme (GoI) in a series of papers. The goal of this programme was to provide a mathematical analysis of the cut elimination process in linear logic proofs. This new interpretation replaces graph-rewriting by information flow in proof-nets. A specific model considered by Girard was based on the C^*-algebra of bounded linear operators on the space ℓ^2. From a computational point of view this yields an analysis of λ-calculus β-reduction and has been applied in such areas as the analysis of optimal reduction strategies. GoI has also brought forward a new perspective for the semantics of computation which places it in a kind of "middle ground" between imperative/procedural, denotational/operational approaches in the semantics of programming languages. This new view helps to model the computational dynamics which is absent in denotational semantics and manages to offer a mathematical analysis which is lacking in operational semantics.

In this talk we give a general algebraic framework for the Girard programme inspired by recent work of Samson Abramsky. We introduce a special class of traced symmetric monoidal categories called *traced unique decomposition categories*. We discuss how to construct models of linear logic based on such categories that we call GoI-models. We also show that such models are *fully complete*, that is completeness with respect to proofs. Next, we define type-free GoI-models and discuss how these provide a GoI interpretation of linear logic proofs. We then complete our journey by going back from type-free GoI-models to discover that we arrive at models of untyped combinatory logic.

DOUG HOWE, Bell Laboratories, Lucent Technologies Murray Hill, New Jersey 07974, USA

Combining functional programming languages and set theory in support of software verification

Computer systems for proving theorems can be coarsely classified as either *automatic* or *interactive*. Automatic systems require theorems to be stated in a simple formalism, such as equational logic or ordinary first-order logic, and use a proof-search procedure that requires little or no guidance from the user. While such systems have produced impressive results in certain branches of mathematics, their proof-search procedures become overwhelmed when proofs require a substantial amount of background knowledge, for example when the provers are used for verifying correctness properties of software (and hardware) systems. Virtually all successful applications of theorem provers to

verification problems have used interactive systems, where a human user is relied upon to provide the main ideas for the proof, and the system handles the trivial details.

Because of the central role of the user in interactive systems, there has been a great deal of research into formalisms that allow users to express their problems and proof ideas as directly and concisely as possible. Some of the formalisms currently in use are *constructive*. These logics are designed to allow direct expression of concepts from programming and to get at the computational content of mathematics. Other formalisms are *classical*, and are used to give conventional mathematical models of software and hardware systems. In this talk, we describe our attempt to unify the two kinds of formalisms. The goal is a logic that supports direct reasoning about programs and abstractions related to programming, and at the same time has the mathematical modelling power of set theory (ZFC). The technical core of our work is a way to combine functional programming languages with conventional concepts from set theory.

FRANCOIS LAMARCHE, INRIA-Lorraine, 54602 Villers-les-Nancy Cedex, France
Spaces for linguistic representations and the semantics of linear logic

There are many areas of computer science that need a "theory of spaces whose points are little spaces themselves". For example in formal language theory (both as a discipline by itself and as an aid to linguistics), the individual words/sentences generated by a formal language are sets-with-structure. In this case the structure is very simple, being only a finite total ordering whose elements are marked by atomic symbols. But this structure, in addition to being variable from word to word (the vectors vary in length), is naturally endowed with a strong spatial character (words are definitely one-dimensional spaces). At the same time the set of all such words/sentences generated by a formal language has a strong intuitive, while hard to formalize, notion of "neighborhood" attached to it. Two sentences may be related by a simple substitution of words, or an active-passive transformation, which shows they are more closely related than two random samples. For another example replace the space of all sentences by the space of all their parsing trees. Here, the main difference is that the spatial character of the "little spaces" is not simply linear anymore, but tree-like.

So we have identified two levels of "spaceness", "big" and "small", the former serving as domain of variation, in the sense of Lawvere, for the latter. It turns out that once it is recognized, this situation appears in many areas of computer science and applied mathematics: concurrency theory (a process is a big space and its little spaces are its states), statistical learning theory, knowledge representation, rewriting theory, population biology...

We will present a general theory of such spaces. It is informed by two paradigms, that have to be adapted to fit together. One is the Grothendieck-Lavwere theory of toposes, with its connection both to geometry and to model theory. The second one is linear logic: The operations that generate and split little spaces will be seen as generalized multiplicative connectors of linear logic, while the structure that unites all the little spaces into a big one proceeds from the additive fragment of linear logic. The natural "cement" between these two paradigms will be seen to be a class of theories in linear universal algebra, which can be seen as a "general theory of little spaces".

JOACHIM LAMBEK, University of McGill
Bilinear logic in linguistics

At least three versions of bilinear (=noncommutative linear) logic have been applied to linguistics. The intuitionistic version (also known as the syntactic calculus), the classical version (recently proposed by Claudia Casadio) and the compact version (proposed by me). Poset models of these deductive systems have been described as residuated monoids, Grishin algerbas and pregroups respectively. A pregroup is a partially ordered group in which each element a has both a left adjoint and right adjoint. In a first approximation to English grammar one works with the free pregroup generated by an ordered set of basic types expressing person, tense, case *etc.* Words are assigned types which are elements of the free pregroups. To check that strings of words are well-formed sentences, only contractions are required. The fact that two left adjoints don't cancel is exploited

for sentences such as "whom did she see?" which otherwise require a Chomskian trace. Information is processed from left to right; thus a hearer will calculate the type of "whom did" before hearing the rest of the sentence. Constraints on WH-transformations are explained by showing that the computation becomes too complicated. For a closer approximation to English grammar one may have to abandon the assumption that the pregroups is free, which is to say that all grammatical rules can be recorporated in the dictionary.

GONZALO REYES, Département de mathématiques, Université de Montréal, Montréal, Québec H3C 3H7
Topics in synthetic differential geometrye

After an introduction to 'synthetic reasoning' used by geometers, physicists and engineers as a motivation for Synthetic Differential Geometry (SDG), some special topics in SDG will be discussed, including categories of differential equations and, more generally, categories of prolongations such as sprays, connections, almost complex structures and the like. The logical basis of SDG will also be discussed.

ROBERT SEELY, Department of Mathematics and Statistics, University of McGill Montréal, Québec, H3A 2K6
Semantics for various noncommutative linear logics

Since Lambek's work on the syntactic calculus in the 1950's, a considerable body of work interpreting logic without structure rules has been developed in terms of monoidal categories. Linear logic is perhaps the best known recent "logic without structure rules", but one structure rule remains in a state of flux in linear logic, namely the exchange rule. Several variants of noncommutative linear logic have been proposed, from the purest noncommutativity (with two variant notions of negation and two notions of implication), the cyclic logic of Yetter (with only one notion of negation, but still two implications), and most recently, a system with both cyclic and commutative connectives due to Abrusci and Ruet. In this talk we shall outline suitable notions of (categorical) semantics for these variants of noncommutative logic, in a modular fashion starting from the AND-OR (or TENSOR-PAR) fragment, based on the notions of linear bicategory and linear functor.

This talk is based on joint work with Robin Cockett, Rick Blute, and Jürgen Koslowski.

ALASDAIR URQUHART, Department of Philosophy, University of Toronto, Toronto, Ontario M5S 1A1
Complexity problems for substructural logics

Substructural logics are typically obtained by restricting the structural rules of contraction and weakening in propositional logic. They include logics (such as relevance logics) that were originally investigated for philosophical reasons, and logics such as linear logic that were inspired by ideas from category theory and computer science.

A surprising feature of these logics is that the simple omission of the structural rules leads in many cases to a drastic increase in complexity. For example, propositional linear logic is undecidable. In other cases, such as linear logic with weakening, the propositional logic is decidable, but of a high intrinsic complexity. In this talk, I shall report on some recent results on the complexity of decidable substructural propositional logics, and explain some of the open questions in the area.

FRANCK VAN BREUGEL, Department of Computer Science, York University Toronto, Ontario M3J 1P3
Towards quantitative verification of systems: a coalgebraic approach

The majority of the verification methods for software systems only produce qualitative information. Questions like "Does the system satisfy the specification?" and "Are the systems semantically

equivalent?" are answered. However, this information is often too restrictive in practice and a (complementary) quantitative approach to verification is needed. For example, answers to questions like "What is the probability that the system satisfies its specification?" and "Do the systems behave almost (up to some small time fluctuations, say of one millisecond) the same?" provide us with (often more useful) quantitative information about the systems.

Metric spaces (and generalizations thereof) seem a good candidate for measuring the difference in behaviour of systems. The behaviour of many software systems can be described by means of coalgebras (of an endofunctor on the category of sets). For most systems, the endofunctor (on sets) associated to the coalgebra can be naturally extended to an endofunctor on metric spaces. This extended endofunctor having a terminal coalgebra is the key to the success of my approach to quantitative verification. The approach will be illustrated by considering restricted classes of real-time and probabilistic systems.

Algebraic Combinatorics, Group Representations and MacDonald Polynomials / Combinatoire algébrique, représentations des nombres et polynômes de Macdonald

(F. Bergeron, N. Bergeron and M. Zabrocki, Organizers)

ED ALLEN, Wake Forest University, Reynolda Station, Winston-Salem, North Carolina 27109, USA

Bitableaux bases for some Garsia-Haiman modules and other related modules

Let $\mathcal{A}$ be the alphabet

$$\mathcal{A} = \{\dots, (0,3), (0,2), (0,1), (0,0), (1,0), (2,0), (3,0), \dots\}.$$

Let $C[X, Y, Z, W]$ be the polynomial ring in the variables $X = \{x_1, x_2, \dots, x_n\}$, $Y = \{y_1, y_2, \dots, y_n\}$, $Z = \{z_1, z_2, \dots, z_n\}$ and $W = \{w_1, w_2, \dots, w_n\}$. Given a subset $S = \{(a_1, b_1), (a_2, b_2), \dots, (a_n, b_n)\}$ of the alphabet $\mathcal{A}$, we define M_S to be the $n \times n$ matrix

$$M_S = (x_i^{a_j} y_i^{b_j})_{1 \le i,j \le n}$$

and $\Delta_S(X, Y)$ to be the determinant of M_S. Let ∂_{x_i} denote the partial differential operator with respect to x_i. With $P(X, Y) \in C[X, Y]$, we will set $P(\partial_x, \partial_Y) = P(\partial_{x_1}, \partial_{x_2}, \dots, \partial_{x_n}, \partial_{y_1}, \partial_{y_2}, \dots, \partial_{y_n})$. Setting $\mathcal{I}_S$ to be the ideal

$$\mathcal{I}_S = \{P(X, Y) \in C[X, Y] : P(\partial_X, \partial_Y)\Delta_S(X, Y) = 0\},$$

we define $R_S(X, Y)$ to be the polynomial quotient ring

$$R_S(X, Y) = C[X, Y]/\mathcal{I}_S.$$

The rings $R_S(X, Y)$ are called the *Garsia-Haiman modules*.

The action of $\sigma \in S_n$ on $P(X, Y, Z, W) \in C[X, Y, Z, W]$ is defined by setting

$$\begin{aligned}\sigma P(x_1, x_2, \dots, &x_n, y_1, \dots, y_n, z_1, \dots, z_n, w_1, \dots, w_n) \\ &= P(x_{\sigma_1}, x_{\sigma_2}, \dots, x_{\sigma_n}, y_{\sigma_1}, \dots, y_{\sigma_n}, z_{\sigma_1}, \dots, z_{\sigma_n}, w_{\sigma_1}, \dots, w_{\sigma_n}).\end{aligned}$$

Set

$$R^+[X, Y, Z, W] = \{P(X, Y, Z, W) \in C[X, Y, Z, W] : \sigma P = P \ \forall \sigma \in S_n\},$$
$$I_{S,T} = \{P \in R^+[X, Y, Z, W] : P(\partial x, \partial y, \partial z, \partial w)\Delta_S(X, Y)\Delta_T(Z, W) = 0\}$$

and

$$R_S^+(X,Y,Z,W) = C[X,Y,Z,W]/I_{S,T}.$$

Analogously, with $\mathrm{sgn}(\sigma)$ denoting the sign of the permutation σ, let

$$R^-(X,Y,Z,W) = \{P(X,Y,Z,W) \in C[X,Y,Z,W] : \sigma P = \mathrm{sgn}(\sigma)P \ \forall \sigma \in S_n\},$$
$$J_{S,T} = \{P \in R^-(X,Y,Z,W) : P(\partial x, \partial y, \partial z, \partial w)\Delta_S(X,Y)\Delta_T(Z,W) = 0\}$$

and

$$R^-_{S,T}(X,Y,Z,W) = C[X,Y,Z,W]/J_{S,T}.$$

We construct bases for $R_S(X,Y)$, $R^+_{S,T}(X,Y,Z,W)$ and $R^-_{S,T}(X,Y,Z,W)$ (for certain generalclasses of S and T that are called *dense*) that are indexed by pairs of standard tableaux and sequences ψ_S and ψ_T.

JEAN-CHRISTOPHE AVAL, Université de Bordeaux I, France
To be announced

FRANÇOIS BERGERON, Université du Québec à Montréal, Montréal, Québec
Diagonal harmonics and generalizations

CAROL CHANG, Department of Mathematics, Northeastern University, Boston, Massachusetts 02115, USA
Representations of quivers with free modules of covariants

A quiver is an oriented graph $Q = (Q_0, Q_1)$ where Q_0 is the set of vertices and Q_1 is the set of arrows. For $\alpha \in Q_1$, $\alpha\colon t\alpha \to h\alpha$. A representation V of a quiver Q is a collection $V = \left\{\left(V_x, V(\alpha)\right) \mid x \in Q_0, \alpha \in Q_1\right\}$ where V_x is a vector space and $V(\alpha)$ is a linear map from $V_{t\alpha}$ to $V_{h\alpha}$. Specifiying a dimension at each vertex of the quiver, a representation is then determined by a point of the affine space $\mathrm{Rep}(Q,\mathbf{d}) = \bigoplus_{\alpha\in Q_1} \mathrm{Hom}_k(V_{t\alpha}, V_{h\alpha})$. There is a natural action of $\mathrm{SL}(Q,\mathbf{d}) = \prod_{x\in Q_0} \mathrm{SL}_{d(x)}(k)$ on $\mathrm{Rep}(Q,\mathbf{d})$.

Given a finite connected quiver Q, we are interested in when the action of $\mathrm{SL}(Q,\mathbf{d})$ on $\mathrm{Rep}(Q,\mathbf{d})$ gives a cofree representation. In particular, we are interested in studying the situation when the modules of covariants are free $k[\mathrm{Rep}(Q,\mathbf{d})]^{\mathrm{SL}(Q,\mathbf{d})}$-modules. We will discuss when quivers have free modules of covariants. We will also discuss the combinatorics involved in describing the orbits of the group action mentioned above.

ADRIANO GARSIA, University of California at San Diego, San Diego, California, USA
An update on the n! conjecture

TUDOSE GEANINA, York University
Littlewood-Richardson rule for a special case of fusion coefficients

The fusion coefficients are the structure constants associated to the fusion algebra of an affine Kac-Moody algebra $\hat{g}$, which can be seen as truncated tensor product coefficients at level k. For $g = A_{n-1}$ Goodman and Wenzl have an equivalent interpretation to the Hecke algebra at root of unity. Using this we are able to give a combinatorial interpretation of these coefficients for the case where one

of the weights is a 2-column partition. In this talk we will describe this interpretation and some applications of it.

VICTOR GINZBURG, University of Chicago
Principal nilpotent pairs in a semisimple Lie algebra

We introduce and study a new class of pairs of commuting nilpotent elements in a semisimple Lie algebra. These pairs enjoy quite remarkable properties and are expected to play a major role in Representation theory. The properties of these pairs and their role is similar to those of the principal nilpotents. To any principal nilpotent pair we associate a two-parameter analogue of the Kostant partition function, and propose the corresponding two-parameter analogue of the weight multiplicity formula. In a different direction, each principal nilpotent pair gives rise to a harmonic polynomial on the Cartesian square of the Cartan subalgebra, that transforms under an irreducible representation of the Weyl group. In the special case of GL_n, the conjugacy classes of principal nilpotent pairs and the irreducible representations of the Symmetric group, S_n, are both parametrised (in a compatible way) by Young diagrams. In general, our theory provides a natural generalization to arbitrary Weyl groups of the classical construction of simple S_n-modules in terms of Young symmetrisers.

MARK HAIMAN, University of California at San Diego, La Jolla, California 92093-0112, USA
The McKay correspondence and the $n!$ conjecture

The McKay correspondence is a remarkable conjecture asserting that if G is a finite group of linear endomorphisms with determinatn 1 of a complex vector space V, and X is a special type (called *crepant*) of resolution of singularities of the orbit space V/G, then the Betti numbers of X sum to the number of conjugacy classes of G. As a step toward explaining the McKay correspondence, Nakamura has proposed that a space known as the G-Hilbert scheme should be a crepant resolution of V/G whenever one exists. When G is the symmetric group acting on the direct sum of two copies of its natural representation, Nakamura's conjecture is equivalent to the "$n!$ conjecture" of Garsia and myself.

LUC LAPOINTE, UdeM
To be announced

ALAIN LASCOUX, Université de Marne La Vallée
Graphe de Yang-Baxter

Nous montrons que de nombreuses proprietes des polynomes de Jack, Macdonald, symetriques ou non, s'annulant ou non, peuvent s'interpreter facilement en terme des relations de Yang-Baxter.

JENNIFER MORSE, University of Pennsylvania, Philadelphia, Pennsylvania 19104 USA
A new basis for Macdonald polynomials

We will present a set of multivariate symmetric polynomials,

$$A_\lambda^{(k)} = \sum_{\mu;\ell(\mu)\leq k} v_{\mu\lambda}^{(k)}(q) S_\mu(x\,;t),$$

with $v_{\mu\lambda}^{(k)}(q)$ a polynomial in q with positive integer coefficients and $\ell(\lambda) \leq k$. We conjecture that for any partition μ with $\ell(\mu) \leq k$, the Hall-Littlewood polynomials can be expanded in this basis as

$$H_\mu(x\,;q,t) = \sum_{\lambda;\ell(\lambda)\leq k} c_{\lambda\mu}^{(k)}(q) A_\lambda^{(k)},$$

where $c^{(k)}_{\lambda\mu}(q)$ is also $\in \mathbb{N}[q]$. The $A^{(k)}_{\lambda}$ basis provides a natural mechanism to divide the set of standard tableaux into families and is loosely related to the atomic decomposition of Lascoux and Schützenberger. We will discuss properties of this basis that are associated to tableaux combinatorics, creation operators, Pieri formulas and the Macdonald polynomials.

SIDDHARTA SAHI, Rutgers University, New York, USA
To be announced

LUC VINET, Université McGill
To be announced

MIKE ZABROCKI, Centre de Recherches Mathématiques, Université de Québec à Montréal, Montréal, Québec H3C 3P8
Special cases of positivity for (q,t)-Kostka coefficients

We present two symmetric function operators H_3^{qt} and H_4^{qt} that have the property $H_3^{qt}H_{(2^a1^b)}[X\ ;\ q,t] = H_{(32^a1^b)}[X\ ;\ q,t]$ and $H_4^{qt}H_{(2^a1^b)}[X\ ;\ q,t] = H_{(42^a1^b)}[X\ ;\ q,t]$. These operators are generalizations of the analogous operator H_2^{qt} and have expressions in terms of Hall-Littlewood vertex operators. The vertex operator formulas are used to give formulas for generating functions for classes of standard tableaux that generalize the case when μ is two columns. This gives statistics, $a_\mu(T)$ and $b_\mu(T)$, on standard tableaux such that the q,t Kostka polynomials are given by the sum over standard tableaux of shape λ, $K_{\lambda\mu}(q,t) = \sum_T t^{a_\mu(T)}q^{b_\mu(T)}$ for the case when when μ is two columns or of the form (32^a1^b) or (42^a1^b). This provides proof of the positivity of the (q,t)-Kostka coefficients in the previously unknown cases of $K_{\lambda(32^a1^b)}(q,t)$ and $K_{\lambda(42^a1^b)}(q,t)$.

Computing and Mathematical Modelling / Calcul et modélisation mathématique
(P. Hansen and G. Laporte, Organizers)

YOSHUA BENGIO,
Learning from structured high-dimensional data

P. W. FOWLER, The fullerenes: interesting graphs and fascinating molecules
School of Chemistry, University of Exeter, Exeter EX4 4QD

This contribution will discuss the new all-carbon fullerene molecules, of which the prototype is the famous C60 football, and show how some very simple ideas of symmetry, geometry and graph theory, together with pictorial bonding theory can be used to count, construct and classify fullerenes, estimate their likely stabilities, follow their interconversion pathways and make predictions about the types and structures of their chemical derivatives. The derived rules of thumb give insight into the physical properties and extensive chemistry of these all-carbon cages.

The lecture will concentrate on showing the many ways in which a little mathematics has already been a great help to chemical and physical understanding of these and related molecules, and will

discuss various directions for joint exploration by mathematicians and chemists.

PIERRE HANSEN AND GILLES CAPOROSSI, GERAD and Ecole des Hautes Etudes Commerciales et Ecole Polytechnique de Montreal
Variable neighborhood search for extremal graphs A. computer-aided search and applications

Finding extremal graphs for some invariant, or for a function of several invariants, possibly subject to constraints, can be viewed as a problem of combinatorical optimization on an infinite familly of graphs. Consequently, powerful metaheuristics may be used to solve this problem. The Variable Neighborhood Search (VNS) metaheuristic, is used within the system AutoGraphiX (AGX), which has many capabilities such as finding counter-examples to conjectures, or suggesting new ones.

The system has been used to study different problems in graph theory, some interesting but unexpected results were found and will be presented here.

GILLES CAPOROSSI AND PIERRE HANSEN, Ecole Polytechnique de Montreal et GERAD and Ecole des Hautes Etudes Commerciales
Variable neighborhood search for extremal graphs B. automated search for relations between graph invariants

The AutoGraphiX system is designed to find (heuristically) extremal graphs for some invariant, possibly subject to constraints. Using parametrisation, a set of presumably extremal graphs may be produced from which some conjectures may be found. We aim here to automate this process, thus making the system completely automated. Three approaches used are to be described here: A numerical one, using principal component analysis in a particular way, a geometric one based on the definition of a convex hull of the graphs in the space of chosen invariants, and an algebraic one based upon recognition of famillies of extremal graphs and known relations for the so found famillies of graphs.

JACQUES DESROSIERS,
The mathematics behind vehicle routing and crew scheduling

P. W. FOWLER, School of Chemistry, University of Exeter, Exeter EX4 4QD
The fullerenes: interesting graphs and fascinating molecules

This contribution will discuss the new all-carbon fullerene molecules, of which the prototype is the famous C60 football, and show how some very simple ideas of symmetry, geometry and graph theory, together with pictorial bonding theory can be used to count, construct and classify fullerenes, estimate their likely stabilities, follow their interconversion pathways and make predictions about the types and structures of their chemical derivatives. The derived rules of thumb give insight into the physical properties and extensive chemistry of these all-carbon cages.

The lecture will concentrate on showing the many ways in which a little mathematics has already been a great help to chemical and physical understanding of these and related molecules, and will discuss various directions for joint exploration by mathematicians and chemists.

LANGIS GAGNON,
R&D activities in computer vision at CRM

MICHEL GENDREAU, CHRISTELLE WYNANTS AND MARTINE LABBÉ,
Network synthesis with non-simultaneous single commodity flow requirements

RONALD M. HARSTAD, RUTCOR and Rutgers Business School, Rutgers University
Developments in automated short-term stockmarket trading

Financial engineers have developed sophisticated models for estimating both values of particular assets, and portfolio balance adjustments. The developments I will report on are less sophisticated from the standpoint of computational design, but are typically being used over much shorter time horizons. An artificially intelligent agent runs parallel processing on 4 NT workstations, and spots trades that are likely to yield a profit in a very short time. Positions are opened without guidance or approval of a human being. Balancing trades closing the positions usually involve human choices over computer-generated alternatives. The typical position is opened and closed in 10–40 seconds. This agent has recently grown to become responsible for over 3 percent of trading volume on the NASDAQ market.

GILBERT LAPORTE, GIANOPAOLO GHIANIK AND FRÉDÉRIC SEMET,
The black and white travelling salesman problem

CHARLES LIN, University of Montreal, Montreal, Quebec
Regional modelling of the atmosphere

We discuss regional modelling of the atmosphere using a high resolution numerical model (MC2: Mesoscale Compressible Community Model). We present an overview of the model, and results from three applications. They are the study of precipitation from Montreal area storms and comparison with values retrieved from the McGill radar, the surface wind and temperature fields over the Gulf of St. Lawrence and comparison with station observations, and a simulation of the severe precipitation of the Saguenay storm that gave rise to flash floods in the Saguenay region in July 1996.

RICHARD LOULOU,
Mathematical models in energy/environment with a special emphasis on the greenhouse gas effect

DOMINIQUE PELLETIER ET JEAN-YVES TRÉPANIER,
Survol des activités du programme de mécanique industrielle du CERCA

MICHAEL SHALMON,
Random walk ladders, branching decompositions and queues

MICHAEL SHALMON,
How fast is the stochastic evaluation of the expected queuing performance and of its gradient

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General History of Mathematics / Histoire générale des mathématiques
(R. O'Lander and R. Sklar, Organizers)

TOM ARCHIBALD, Acadia University, Wolfville, Nova Scotia B0P 1X0
Mathematics in France, 1870–1890: A view via doctoral theses

The period immediately following the French defeat by Prussia in 1870–71 was one of profound transformation for French mathematics. This paper examines the influence of German mathematical work in France during this period, with particular attention to the evidence provided by the theses of the seventy-four mathematicians receiving a doctorate during this interval. Many of these writers remain very well-known to this day (such as Poincaré, Picard, and Painlevé) or were highly regarded at the time (Floquet, Halphen, Humbert, *etc.*); others, while more obscure, were of historical importance via their teaching and organizational activity. On the one hand, we find that the many theses associated with the research interests of Charles Hermite exercised a very strong influence on the subsequent direction of French mathematical research. On the other hand, these theses incorporate a rich body of German work from writers such as Weierstrass, Kronecker, Fuchs, Riemann, Clebsch, Schwarz, and Carl Neumann.

EDWARD BARBEAU, University of Toronto, Toronto, Ontario M5S 3G3
Bringing history close to home: Pell's equation

An excellent topic for acquainting students with the historical flow of mathematical discovery is Pell's equation. For given integer parameters d and k, one seeks integer solutions for $x^2 - dy^2 = k$. Natural questions about numbers, which students can find and investigate themselves, lead to this equation. They can match wits with early Indian and European mathematicians in tackling them. We can trace the evolution from ad hoc (albeit ingenious) to systematic techniques, and follow the progress to a theoretical framework for the mathematical structure of the set of solutions. This is an area that can be handled at many levels of sophistication.

LILIANE BEAULIEU, Centre de recherches mathématiques, Universit de Montréal Montréal, Québec
Clips from Bourbakian skits

Discussions about the overall plan of Bourbaki's *Éléments de mathématique* were led, for many years, by Jean Dieudonné, who at nearly every meeting submitted an outline to the critical eyes of his peers. In the as yet unpublished reports of the meetings, one reads that the members usually reacted to the "overall-plan act" as to a comical skit, mobbing the actor who brandished the prospect of hard toil. This paper studies the overall plans which were written for Bourbaki between 1935 and 1956 as evidences of some major turns in the early Bourbaki enterprise.

LEN BERGGREN, Simon Fraser University, Burnaby, British Columbia V5A 1S6
Three geometrical gems from medieval Islamic mathematics

The geometer Abu Sahl Al-Kuhi, who worked in the latter half of the tenth century A.D., excelled in solving problems arising from some of the great traditions of ancient geometry—notably those represented by some of Archimedes works. However, among his extant works are several short ones dealing with problems that do not address any of the famous problems of Greek geometry, but are, nonetheless, intriguing. We shall summarize the contents of these geometrical gems and fit them into the context of the mathematics of the time.

STANLEY BURRIS, University of Waterloo, Waterloo, Ontario
Reconstructing Boole's discovery of an algebra of logic

Boole developed an algebra of logic that certainly was not Boolean algebra or Boolean rings. His two texts give little indication of how he arrived at his system. In this talk an attempt is made to provide a framework that makes the development of Boole's ideas appear to be natural.

JAL CHOKSI, Department of Mathematics and Statistics, McGill University, Montreal, Quebec H3A 2K6
A history of the convergence theorems of (Lebesgue) integration

In most courses on Lebesgue integration there are three main convergence results: (i) the monotone convergence theorem (MCT), (ii) Fatou's lemma, both for non-negative functions, (iii) the dominated convergence theorem (DCT) and its corollary (for finite total measure) the bounded convergence theorem (BCT). They are most often proved in that order. DCT is the most often used in practice. Historically, things were very different! Lebesgue's thesis (1902) contains only BCT. It was only 4 years later, in 1906, that Beppo Levi proved MCT and independently Fatou proved his lemma. The proofs, each starting with BCT, are very similar. DCT first appears in a paper of Lebesgue in 1908, with a more detailed account in a paper in 1910. The proof is similar to his original proof of BCT. Earlier, in 1907, Vitali had proved a convergence theorem using the concept of uniform absolute continuity of the integrals, we shall discuss this and it's subsequent use. We shall also talk about the work of F. Riesz and Fischer on L^2 convergence (starting around 1905), and Riesz' subsequent generalization to L^p, but this history is better known. The best reference book is Hawkins, Lebesgue's theory of integration, but our lecture may contain a few surprises, even to those who have read this book!

SUH CHUN CHONGS, Athabascau University, Athabasca, Alberta T9S 3A3
Historical background for sequences and calculus

¿From the works of Newton and Leibniz on sequences and series, we try to find the idea for The Fundamental Theorem of Calculus.

FLORIN DIACU, Department of Mathematics and Statistics, University of Victoria, Victoria, British Columbia V8W 3P4
A central long loop: celestial mechanics → algebraic topology → celestrial mechanics

Most mathematical theories are like unfaithful offspring: they forget their origins. But some remember them. In 1892, while pursuing his studies on the 3-body problem, Poincaré laid the foundations of algebraic topology. The new field flourished, finding applications in many branches of mathematics. A hundred years later its tools were used to answer Poincaré's initial question. This talk will present the history and consequences of this problem.

HARDY GRANT, York University, North York, Ontario M3J 1P3
Greek mathematics in cultural context

The often substantial prestige and influence enjoyed by mathematics through the long history of western culture can be traced to beginnings in ancient Greece. Here arose, for example, the vision of mathematical knowledge both as uniquely certain—because attained by rigorous proof from incontestable axioms—and as potentially allowing unique insight into the cosmic order. I shall try to sketch both (i) the development, in cultural context, of the mathematical tradition that culminated in Euclid's exemplary *Elements*, and (ii) the concomitant influence of contemporary mathematics on such characteristically Greek achievements as Plato's theory of Forms, Aristotle's theory of scientific

method, and the "liberal arts" tradition in education.

MINORU HASEGAWA, Department of Computer Science, Lakehead University, Thunder Bay, Ontario P7B 5E1
From data manipulation to data exploration—a new frontier

An explosive advancement of network systems and computing power has brought us powerful tools to search and extract valuable information from (seemingly unrelated)sets of (NOT-so-well-defined) data. It is now possible to identify an individual INSTANCE (datum) which satisfies a required set of conditions out of huge (hostile) data. DATA MINING and other related disciplines seem to emerge as a new frontier.

NORBERT HODGSON, Département de mathématiques et de statistique, Université Laval, Laval, Québec G1K 7P4
Histoire des mathématiques et formation des enseignants du secondaire : une expérience d'utilisation de textes originaux

L'exposé portera principalement sur des aspects pédagogiques reliés à l'histoire des mathématiques et comprendra deux parties. Dans un premier temps, je présenterai un cours d'histoire des mathématiques créé r'ecemment par mon département à l'intention des futurs enseignants du secondaire, cours distinct de celui déjà offert aux étudiants de mathématiques. Je m'intéresserai ensuite à la question de l'utilisation de textes originaux dans un tel cours et ferai état d'une expérience réalisée à partir du "Liber quadratorum" (1225) de L'eonard de Pise (dit Fibonacci).

G. MOORE, McMaster
The early reception of Klein's Erlanger programme: 1872–1916

RICHARD O'LANDER, St. John's University, Jamaica, New York 11439, USA
The new math a look back

Today there is a great outcry from parents, teachers, business people, politicians and others about the low academic performance of many of our high school graduates. This is especially true when it comes to mathematics. "How will we compete with the Japanese and German's?" is the new rallying cry. This is not the first time such a concern has been raised. The 1950's and 1960's were a time of great curriculum reform in the pre-college mathematics. The reform movement was imitated in part by the supposed "technological gap" between the United States and the Soviet Union. The purpose of this paper is to discuss the basic premise behind the "New Math", as well as its successes and failures.

NORBERT SCHLOMIUK, Department of Mathematics and Statistics, University of Montreal, Montreal, Quebec H3C 3A7
Andre' Weil (1906–1998), in memoriam

La confe'rence portera sur la vie d'Andre' Weil, un des grands mathe'maticiens de notre sie̯le. Andre' Weil et l'histoire des mathe'matiques.

RONALD SKLAR, St. John's University, Jamaica, New York 11439, USA
Computational logic: 1950's–1965

The idea of mechanizing mathematics can be traced back to Descartes and Leibniz. But the first truly automated proofs in mathematics had to wait until the 1950s and the invention of the electronic

computer. The purpose of this talk is to trace the use of logic in automated deduction with particular emphasis on the use of the principle of resolution. Along the way the contributions and ideas of Descartes, Leibniz, Skolem, Herbrand, Hilbert and Ackermann, Gödel, Church, Turing, Davis and Putnam, J. A. Robinson and others will be briefly discussed.

VIENA STASTNA, Department of Mathematics, University of Calgary, Calgary Alberta T2N 1N4
Math caught a wedding bouquet. Young years of Sonja Kovalevskaja

Sonja and her "nihilist" sister—girls of sixties. University classes only for married women. Ph.D. from Gottingen after private lessons with Weierstrass. As a young widow—a position at Stockholm University. 1888—Prix Bordin from Paris Academy of Sciences. Dead at 41.

GEORGE P. H. STYAN, Department of Mathematics and Statistics, McGill University, Montreal, Quebec H3A 2K6
Some remarks on five mathematicians and on at least three postage stamps all associated with determinants

The five mathematicians, who were all interested in determinants are: Takakazu Seki k$\hat{(}$0) wa: 1642( ) (?)–1708 Gottfried Wilhelm von Leibniz: 1646–1716 William Spottiswoode: 1825–1883 Charles Lutwidge Dodgson: 1832–1898 Sir Thomas Muir: 1844–1934.

Postage stamps have been issued in honour of Seki, Leibniz and Dodgson.

This is a joint work with R. William Farebrother (Manchester) and Shane T. Jensen (Harvard).

PETER ZVENGROWSKI, Department of Mathematics and Statistics, University of Calgary Calgary, Alberta T2N 1N4
Vector analysis and the great 1890's controversy

Vector analysis has a long history going back even to Greek mathematics, but the foundations for the subject as we know it today were laid in 1843, quite independently and from different points of view, in Ireland (W. R. Hamilton) and in Germany (H. Grassmann). The talk will trace these beginnings, through the amazing controversy they led to in the 1890's, to developments in the 20-th century that to some extend seem to be not very well known in mathematical circles.

Some of the associated pedagogical questions will also be discussed.

Graduate Student Seminar / Seminaire des étudiants aux cycles supérieurs
(P. Libbrecht, T. Mattman and S. Thomas, Organizers)

LEO BUTLER, Department of Mathematics, Queen's University, Kingston, Ontario K7L 3N6
New examples of integrable geodesic flows

A geodesic flow on T^*M^n is said to be *integrable* if it possesses n independent commuting first integrals. Taimanov has proven that if these first integrals are real analytic, then $\pi_1(M)$ is almost abelian and $H^*(M)$ possesses a subring isomorphic to $H^*(T^d)$ where d is the first Betti number of M.

It will be shown that both conclusions are false in the smooth category.

YIN CHEN, Laval
Estimation de spectre pour les élements algébriques

Soit A un algébre de Banach avec unité. Soient $a, b \in A$ deux élements algébriques de degré au plus n. On vais montrer, dans cet exposé, que la distance de Hausdorff entre les deux spectres $\sigma(a)$ et $\sigma(b)$ vérifie une estimation suivante:

$$\Delta\big(\sigma(a), \sigma(b)\big)^n \leq \Big(\frac{2}{3}n + \frac{1}{3}\Big)(2M)^n \|a - b\|,$$

où $M = \max(\|a\|, \|b\|)$. Ce résultat généralise celui de Friedland pour la matrice, et de plus améliore la borne de constante.

THOMAS HUGH, Department of Mathematics, University of Chicago, Chicago, Illinois 60637, USA
A bijection between NBB sets of atoms and descending chains in admissible lattices

A finite graded lattice is admissible if it has a labelling of its join-irreducibles with certain good properties. This labelling induces an edge-labelling, and it is known that the Mobius function of the lattice can be computed in terms of the numbers of maximal chains in the lattice which are descending relative to the edge-labelling. So far, this is based on papers of Stanley's from the 1970s. Recently, Blass and Sagan introduced the idea of NBB sets of atoms of a lattice, and showed that the Mobius function of a lattice could be computed in terms of them. We construct a bijection between the descending chains and the relevant collection of NBB sets. Also, we define quasi-admissibility of a (not necessarily graded) lattice, a generalization of admissibility, for which the above bijection also holds. We show that any lattice with a maximal left-modular chain is quasi-admissible.

HÀ HUY TÀI, Queens University
Box-shaped matrices and their ideals of 2×2 *minors*

The theory on ideals of minors of a matrix has been well studied in many decades. In this talk, I will extend this notion to ideals of minors of "higher dimensional" matrices. The notion of a box-shaped matrix and its minors will be introduced. I will also discuss the geometric realization of these ideals. It turns out that they are the defining ideals for Segre embedding of product spaces.

MIKHAIL KOTCHETOV, Memorial
Identities in Hopf algebras

1) a brief introduction to Hopf algebras (definitions and basic examples), 2) identities and coidentities (definition and basic properties), 3) examples of Hopf algebras with an identity or a coidentity. The notion of a coidentity is new, it is introduced in my paper (see below). As examples of Hopf algebras with an identity or a coidentity Taft's algebras and smash products are considered. For smash products the necessary and sufficient condition of having a polynomial identity is proved. For Taft's algebras the basis of identities and coidentities is found.

References

On Identities for Coalgebras and Hopf Algebras. Comm. Algebra, to appear in early 2000.

RAMIN MOHAMMADALIKHANI, Department of Mathematics, University of Toronto, Toronto, Ontario
To be announced

I will give an introduction to the theory of moduli spaces of flat connections on a Riemann surface and the role of symplectic geometry in that. There will be an emphasis on the physical motivation of the subject.

No original work is to be presented. It is just an attempt to show to the other graduate students what this subject is about, what some of its goals are, as many students might have already heard some titles and they might be curious to know more about that.

NATHAN NG, University of British Columbia
Prime number races

In 1853, Chebyshev observed that there seemed to be more prime numbers congruent to 3 mod 4 than to 1 mod 4. In 1994, Micheal Rubinstein and Peter Sarnak were able to explain this phenomenon under certain natural hypotheses concerning Dirichlet L-functions. I will describe generalizations of their results to Chebotarev's Density Theorem. Chebotarev's density theorem is a theorem from algebraic number theory that describes certain interesting sets of prime numbers. Numerical results presented will depend on having large lists of zeros of Artin L-functions.

MARTIN PERGLER, University of Chicago, Chicago Illinois 60637, USA
Connection preserving actions and observable and epimorphic subgroups

Let G be a real algebraic group. We consider which (algebraic) subgroups H arise as point stabilizers in affine connection preserving G-actions on manifolds, and for which H are any H-fixed points of such actions necessarily G-fixed points.

We prove that under certain hypotheses (conjecturally always) these are the same H with analogous properties concerning invariant vectors in (finite-dimensional) linear representations, called observable and epimorphic subgroups. Techniques involve contrasting local dynamics and linearization of the action near fixed points, together with the structure theory of epimorphic subgroups. The result forms a part of Zimmer's program of studying representations of Lie groups into the automorphism groups of geometric structures on manifolds.

We also extend results of Bien and Borel to classify all epimorphic subgroups of SL_n normalized by a maximal torus, in terms of graphs on n vertices and a certain subset of the Tits boundary.

SHALOUB RAZAK, Department of Mathematics, University of Toronto Toronto, Ontario M5S 3G3
Classifying C^*-algebras

What are C^*-algebras? How are they classified? During the last decade the program to classify amenable C^*-algebras has been a very active field of mathematical research. I will survey some of the high points, and briefly describe my contribution to the classification program.

DOMINIC ROCHON, Universit/'e de Montréal, Montréal Québec H3C 3J7
Dynamique bicomplexe

Les nombres bicomplexes sont une généralisation des nombres complexes définie à l'aide d'une multiplication commutative. Cette conférence aura pour but de vous introduire à la généralisation de l'ensemble de Mandelbrot pour les nombres bicomplexes.

KONE SIAKA, Department of Mathematics, University of the Witwatersrand, Wits 2050, Johannesburg, South Africa
Mixed order systems of ordinary linear differential equations

We have characterized a sufficiently large classes of mixed order systems of differential equations which are equivalent to first order systems. This has been shown through the reduction of mixed order systems to first order systems. Writing mixed order systems in operator form, we have shown that this operator is equivalent to an operator for associated first order systems.

GREGORY G. SMITH, Mathematics Department,University of California, Berkeley, California 94720, USA
Initial ideals in the Weyl algebra

Let $A_n(k)$ be the Weyl algebra over a field of characteristic zero and let M be a finitely generated left $A_n(k)$-module. If $A_n(k)$ is equipped with a filtration such that the associated graded algebra is the commutative polynomial ring in $2n$ indeterminates, we prove that each irreducible component of the characteristic variety of M has dimension at least n. In particular, this generalizes an important consequence of the fact that characteristic variety is involutive when $A_n(k)$ has the order filtration. We also establish, for certain skew polynomial rings including the Weyl algebra and universal enveloping algebras for finite dimensional Lie algebras, an equidimensionality theorem for the characteristic variety, extending known results to non-Zariskian filtrations.

MICHAEL SOLTYS, Department of Mathematics, University of Toronto, Toronto, Ontario
Boolean programs and quantified propositional proof systems

My talk is going to be based on the following article: *Boolean Programs and Quantified Propositional Proof Systems*, which I coauthor-ed with Steven Cook, and which is going to appear this Fall in the *Bulletin of the Section of Logic*.

In this article we introduce the notion of Boolean programs, which provide more concise descriptions of Boolean functions than Boolean circuits. We characterize nonuniform PSPACE in terms of polynomial size families of Boolean programs. We then show how to use Boolean programs to witness quantifiers in the subsystems G_1 and G_1^* of the proof system G for the quantified propositional calculus.

ADAM VAN TUYL, Queens
Computing the spreading and covering numbers

Let $S = k[x_1, \ldots, x_n]$, d a positive integer, and suppose that $S_d := \{m_1, m_2, \ldots, m_l\}$ where $l = \binom{d+n-1}{n-1}$ is the set of all monomials of degree d. Let $V \subseteq S_d$ be a subset of monomials and define $s(n,d) := \max\{\dim V | \dim S_1 V = n \dim V\}$ and $c(n,d) := \min\{\dim V | S_1 V = S_{d+1}\}$. The numbers $s(n,d)$ and $c(n,d)$ are called the *spreading numbers and covering numbers*, respectively. These numbers are of interest because of their connection to the Ideal Generation Conjecture. We describe a new approach to calculate these numbers that uses simplicial complexes. This is joint work with Tai Ha of Queen's University and Enrico Carlini of the University of Genova.

KHALID EL YASSINI, Sherbrooke
Analysis of two interior-exterior penalty algorithms for linear programming

We describe two interior-exterior algorithms for linear programming problem. The algorithms are based on path following idea and use a two parameter mixed penalty function. Each iteration updates the penalty parameters. An approximate solution, of Karush-Kuhn-Tucker system of equations which characterizes a solution of the mixed penalty function, is computed by using only one Newton direction in the first algorithm and by predictor-corrector method in the second algorithm. The approximate solution obtained gives a dual and a pseudo-feasible primal points. Since the primal solution is non feasible, a new pseudo-gap definition is introduced to characterize primal and dual solutions. Finally, Some numerical results will be presented.

Mathematical Physics—2 sub-sessions / Physique mathématique—2 sous-sessions
(G. Bluman, M. Grundland and G. Slade, Organizers)

I) Probability Methods and Applications / Méthodes probabilistes et applications

CHRISTIAN BORGS, Microsoft Research
Partition function zeros for first order phase transitions

It is well known that phase transitions are closely related to zeros in the complex temperature and field plane. In this talk I present a new approach to determine partition function zeros near first order phase transitions. I discuss the relation of our results to the Lee Yang theorem on the one hand, and to the theory of finite size scaling on the other.

ALMUT BURCHARD, Department of Mathematics, University of Virginia, Charlottesville, Virginia 22903, USA
Minimal and random spanning trees in two dimension

This talk will describe joint work with M. Aizenman, C. Newman, and D. Wilson, on the subject of (continuum) scaling limits for stochastic tree processes. Three examples will be discussed in some detail:

(1) The uniformly random spanning tree on a planar square lattice,

(2) the minimal spanning tree on a planar square lattice with random edge weights,

and

(3) the Euclidean (minimal) spanning tree on a Poisson point process in the plane.

The infinite-volume limit is known to exist in each case. Here, we consider scaling limits where the typical distance between neighboring sites is taken to zero.

As a first step, we construct a common configuration space, which is analogous to a space of curves. The formulation of the tree processes as probability measures on this space remains meaningful in the scaling limit. Scaling limits exist—at least along suitable subsequences—by a compactness argument. Furthermore, we describe some basic properties of the limiting measures, such as bounds on the dimension of tree branches, and bounds on the number and degree of branching points. The main step in the proof of these statements is to establish a family of scale-invariant bounds on the probability of repeated crossings of annuli by the random tree. The bounds are verified separately for each of the three models.

NEAL MADRAS, Department of Mathematics and Statistics, York University, Toronto, Ontario M3J 1P3
Self-avoiding walks with drift

The self-avoiding walk has long been used as a lattice model of a long polymer molecule in a good solvent. We present a model of a self-avoiding walk in which steps in different directions have different probabilities. This can model a type of polymer that is oriented by an external magnetic field. It can also be used in a model of flux lines in superconductors. This talk will introduce some of the physics background and describe some physically relevant mathematical results.

This is joint work with Christian Borgs, Jennifer Chayes, and Christopher King.

JEREMY QUASTEL, Department of Mathematics and Statistics, University of Toronto, Toronto, Ontario M5S 3G3
Hydrodynamic limits

We will survey progress on the fluid dynamical behaviour of stochastic lattice gases.

MARY BETH RUSKAI, Department of Mathematics, University of Massachusetts Lowell, Lowell, Massachusetts 01854, USA
Pauli exchange errors in Quantum computation

The Pauli exclusion principle implies that fault-tolerant quantum computation requires the ability to correct a special type of two-qubit error which is equivalent to exchanging the qubits. We give an explicit (non-additive) 9-bit code which can handle both Pauli exchange errors and all one-bit errors.

YVAN SAINT-AUBIN, Département de mathématiques et de statistique, Université de Montréal, Montréal, Québec H3C 3H7
Boundary states for a free boson defined on finite geometries

Langlands recently constructed the map $\varphi \to |x(\varphi)\rangle$ that factorizes the partition function of a free boson on a cylinder with boundary condition given by two arbitrary functions φ_{B_1} and φ_{B_2} in the form $\langle x(\varphi_{B_2})|q^{L_0+\bar{L}_0}|x(\varphi_{B_1})\rangle$. We rewrite $|x(\varphi)\rangle$ in a compact form, getting rid of technical assumptions necessary in his construction. This simpler form allows us to explore the properties of the map $\varphi \to |x(\varphi)\rangle$ under conformal transformation that preserve the boundary. (Joint work with Marc-André Lewis.)

GORODON SLADE, Montreal
To be announced

II) Group Theory Methods and Application / Méthodes de la théorie des groupes et applications

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STEPHEN ANCO, Department of Mathematics, Brock University, St. Catharines, Ontario L2S 3A1
Conservation laws of field equations

A computationally effective method for finding the local conservation laws of PDEs is presented using adjoint symmetries (*i.e.* the solutions of the adjoint equation of the determining equation for symmetries). It is shown that for any given PDE(s), all local conservation laws can be constructed by an integral formula in terms of adjoint symmetries satisfying certain conditions. The effectiveness of this approach lies in being able to calculate these adjoint symmetries as solutions of a system of extended determining equations (*i.e.* the adjoint symmetry determining equation plus the conditions) working just on the solution space of the PDE(s) by an algorithmic procedure similar to calculating symmetries. Some applications to classifying conservation laws of various field equations

in mathematical physics are given.

P. BRACKEN, Université de Montréal and McGill University
The generalized Weierstrass system for mean curvature surfaces and the completely integrable sigma model

The integrability of a system which describes constant mean curvature surfaces by means of the generalized Weierstrass inducing formula is studied. This is carried out by using a specific transformation which reduces the initial system to the completely integrable two-dimensional Euclidean sigma model. A new linear spectral problem equivalent to the generalized Weierstrass system is derived via the method of differential constraints. Furthermore the Auto-Bäcklund transformation for the generalized Weierstrass system can also be constructed. The permutability theorem for this Auto-Bäcklund transformation is formulated. New classes of non-splitting multi-soliton solutions are obtained by exploiting this Auto-B"acklund Transformation. A technique for reduction of generalized Weierstrass system to decoupled linear equations by subjecting it to certain differential constraints is presented as well.

JOHN HARNAD, Concordia University
The bilinear differential equations satisfied by Fredholm determinants appearing in random matrices

Bilinear differential equations satisfied by the Fredholm determinants of certain integral operators appearing as spectral distribution functions for random matrices are deduced from the associated systems of nonautonomous Hamiltonian equations satisfied by auxiliary canonical phase space variables introduced by Tracy and Widom. The essential step is to express the latter as isomonodromic deformation equations for families of rational covariant derivative operators on the Riemann sphere and to interpret the Fredholm determinants as isomonodromic τ-functions.

NIKY KAMRAN, Department of Mathematics and Statistics, McGill University, Montreal, Quebec H3A 2K6
Non-existence of time-periodic or quasi-periodic solutions of the Dirac operator in stationary axisymmetric black hole geometries

We have recently proved that the Dirac equation does not admit time-periodic or quasi-periodic solutions in the maximal analytic extension of the non-extreme Kerr-Newman charged rotating black hole. These theorems indicate that in contrast with the classical case of charged massive particle orbits, a quantum mechanical Dirac particle must either fall into the black hole or escape to infinity. We will outline the proofs of these results in our talk. This is joint work with F. Finster, J. Smoller and S.-T. Yau.

A. KOUDRIAVTSEV, No affiliation
To be announced

FRANÇOIS LALONDE, Universite du Québec à Montréal
Vers une geometrie intrinseque sur le groupe des diffeomorphismes hamiltoniens et ses relations avec la cohomologie quantique

Il y a quelques annees, Hofer a decouvert une nouvelle geometrie bi-invariante (et donc intrinsequement definie, independante des choix arbitraires de coordonnees) sur le groupe des diffeomorphismes hamiltoniens d'une variete symplectique quelconque. Il se trouve que cette geometrie est unique, et que pour cette raison, elle place dans un eclairage nouveau les relations entre mecanique classique

et mecanique quantique. On montrera comment cette geometrie est liee au developpement (a venir) d'une theorie Yang-Mills de type L^∞.

MARTIN LÉGARÉ, Mathematical Sciences, University of Alberta, Edmonton, Alberta
Low dimensional integrable systems and topological Yang-Mills theories

A set of integrable systems in 1 or 2 dimensions are shown to be associated to BRST invariant systems derived from topological gauge theories. These integrable systems are also spinor extended to systems related to non-Abelian monopoles equations. Solutions and residual topological actions have been obtained.

JIRI PATERA, Université de Montréal
Non-standard exploitation of cut and project point sets ("Quasicrystals for cryptography")

Cut and project sets are deterministic sets of points with the Delone property in a real Euclidean space which contain no periodic subsets. The original motivation for their invention came from modelling of physical quasicrystals, (equivalently 'aperiodic crystals') in 1, 2, and 3 dimensions. Many of their properties were described in recent years, in particular the fact that they can be generated using simple and very fast algorithms with absolute precision. In the talk some basic properties of such point sets will be recalled and their exploitation for construction of random number generators and masking sequences in symmetric key cryptography will be suggested.

GREGORY REID, Department of Applied Mathematics, University of Western Ontario London, Ontario
Deformations and symmetries of nonlinear differential systems

Symbolic analysis, symmetries and deformations of differential systems are examined in several contexts.

Deformations of differential systems are related to the differential index which is a measure of the stability of the numerical solution of constrained systems of differential equations.

Nonlinear differential Lie systems are deformed to their corresponding linear differential systems by linearization about the identity map. Here the group is deformed to its Lie Algebra, with the homotopy from the Lie algebra to the Lie group being the exponentiation map.

Deformations under the toral group are scalings of the independent and dependent variables of differential systems. Nonlinear differential systems invariant under this group are classified, and their properties explored, in analogy to the Grobner Deformations of Sturmfels.

Some open questions regarding deformation as a generalization of the symmetry concept are also discussed.

PAVEL WINTERNITZ, Université de Montréal
Nonlinear differential and difference equations with superposition formulas

A class of nonlinear ordinary differential equations exists for which the general solution can be written as a function of a finite number of particular solutions. The equations and the superposition formulas are based on transitive effective Lie group actions on homogeneous manifolds. The equations can be discretized to obtain difference equations that also allow superposition formulas. These equations are linearizable by an embedding into a higher-dimensional space. They occur as Backlund transformations in the theory of integrable systems.

Mathematical Genetics and Genomics / Génétique et génomique mathématiques
(S. Lessard and D. Sankoff, Organizers)

KEVIN ATTESON, Mathematical Sciences Research Institute, 1000 Centennial Drive, Berkeley, California 94720-5070, USA
Identifiability and consistency in phylogenetics

In 1978, Joe Felsenstein demonstrated that the popular parsimony method of phylogenetics is statistically inconsistent, that is, that it need not converge to the true tree as amount of data goes to ∞, for a simple stochastic model of the mutation of genetic sequences. Since then, the consistency question has been answered for numerous methods under a variety of stochastic mutation models. When mutation rates vary along the genetic sequence according to a completely unknown distribution, Steel et. al. have shown that the phylogenetic tree is unidentifiable, that is, that there exists no consistent algorithm. The author intends to briefly review results in this area, presenting his own results in this context, namely, the consistency of neighbor-joining methods and the identifiability of the phylogenetic tree when rates are distributed according to a Gamma distribution.

ANDREAS DRESS, University of Bielefeld, Germany
Cluster analysis and phylogenetic nets

Over the past fifteen or twenty years, it has become evident that phylogenetic relationships cannot always be represented appropriately by one single phylogenetic tree—sometimes because too many trees look almost equally plausible, sometimes just because of reticulated evolution.

To deal with data that do not support a single tree, standard tree-building methods have to be replaced by methods that allow for reticulation and, hence, construct (if necessary) rather nets than trees. A good starting point for developing such a method is the observation that Isbell's construction of injective hulls of metric spaces leads to trees if and only if one applies it to metrics that fit isometrically into (R-)trees, and leads to delta-hyperbolic, contractible, polyhedral cell complexes of higher dimension if this is not the case.

For this reason, Isbell's construction has been analysed and, actulally, scrutinized within the context of "overlapping" clustering for more than ten years by now, resulting in quite a few publications dealing with various aspects of this construction.

In the lecture, the basic definitions will be reviewed, new results (many of which have been obtained jointly with Katharina Huber and Vincent Moulton) will explained, and consequences for phylogenetic analysis will be discussed.

R. C. GRIFFITHS, Department of Statistics, University of Oxford, Oxford, United Kingdom OX1 3TG
Ancestral inference from gene trees

A unique gene tree describing the mutation history of a sample of DNA sequences can be can be constructed as a perfect phylogeny under an assumption of non-recurrent point mutations. The tree is equivalent to the DNA sequence data and because of ancestry there is much interest in thinking of the data as a tree.

The likelihood of a gene tree under a stochastic coalescent model of evolution can be found by an advanced simulation technique of importance sampling on genealogies, thus allowing maximum likelihood estimation of parameters using the full information in the data.

The distribution of the time to the most recent common ancestor, ages of mutations in the gene tree, and other ancestral characteristics conditional on the gene tree topology can also be found by

the simulation technique.

TAO JIANG, Department of Computer Science, University of California, Riverside California 92521, USA
Quartet cleaning: efficient algorithms and simulations

A critical step in all quartet methods for constructing evolutionary trees is the inference of the topology for each set of four species (*i.e. quartet*). It is a well-known fact that all quartet topology inference methods make mistakes that result in the incorrect inference of quartet topology. These mistakes are called *quartet errors*.

In this talk, I first give an introduction to general paradigm of reconstructing evolutionary trees based on quartets. Then some efficient algorithms for correcting bounded numbers of quartet errors are presented. These "quartet cleaning" algorithms are shown to be optimal in that no algorithm can correct more quartet errors. An extensive simulation study reveals that sets of quartet topologies inferred by three popular methods (Neighbor Joining, Ordinal Quartet and Maximum Parsimony) almost always contain quartet errors and that a large portion of these quartet errors are corrected by the quartet cleaning algorithms.

MING LI, University of Waterloo, Waterloo, Ontario
Whole genome phylogeny

We are witnessing a revolution in molecular biology and medicine: complete genomes for dozens of species are already available; genomes for many other species, including the *H. sapiens*, will be completed very soon.

Analyzing such information becomes one of the most interesting questions in bioinformatics research. In this talk, we will present a theory based on Kolmogorov complexity, and an actual program implementing this theory, for measuring the *distance* between two genomes. Using these, we are able to construct evolutionary trees (consistent with known trees) from complete genomes. Our new algorithm does not need multiple sequence alignment, does not suffer from the problem of getting different trees for different genes, and uses the complete information of a genome. The program depends on efficient compression of DNA sequences. We will also present a compression program with best compression ratios (on all benchmark sequences) for DNA sequences.

NICHOLAS SCHORK, Department of Epidemiology and Biostatistics, Case Western Reserve University, Cleveland, Ohio; Program for Population Genetics, Department of Biostatistics, Harvard School of Public Health, Boston, Massachusetts; The Jackson Laboratory, Bar Harbor, Maine; Currently on leave sponsored by The Genset Corporation, La Jolla, California.
The future of genetic case-control studies

The power and simplicity of the case/control study design makes it an appealing strategy for testing the relationship between particular genetic polymorphisms (or haplotypes) and traits or diseases. Despite this fact, there are a number of problems that plague the use and interpretation of genetic case/control studies. We describe methods that should allow researchers to evaluate and in many instances overcome many of these problems. These methods rely on the use of multiple genetic markers and can be used to investigate and control for the following problems: 1. genetic stratification; 2. allelic and locus heterogeneity; 3. indirect association and linkage disequilibrium; 4. sample heterogeneity; 5. the assessment of statistical significance; 6. the evaluation of the power and likely yield of a case/control study; and 7. The assessment and use of admixture. We showcase these methods with actual data and argue that because of its simplicity, the case/control design may

become a design of choice rather than a design by default for future genetic epidemiology studies.

KATY L. SIMONSEN, Statistics Department, Purdue University, West Lafayette, Indiana 47905-1399, USA
Probability models for genetic factors underlying a binary phenotype

Probability models in quantitative genetics have a long history. Models for linkage between a single binary trait locus (BTL) and a single genetic marker, and for two or three BTL and marker loci have been developed for specific experimental designs such as backcross and F_2. However, these models do not generalize easily to other designs, and do not allow for varying penetrance with multiple BTL. The development of a more general model with explicit parametrization for incomplete penetrance and multiple genes would permit the expansion of detection methodology into other experimental designs. In this work a general probability model for linkage between an arbitrary number of BTL and marker loci is developed. This model incorporates varying penetrance, and allows for any experimental design as long as the gametic frequencies of the originating parental lines are known. The generality of the formulation streamlines the computer implementation of statistical methods for locating BTL. The model easily reduces to well known special cases, and the joint probability distribution can be used to generate expected values and conditional probabilities for any experimental population derived from the initial cross of two different inbred parental lines. This model allows further practical developments for estimation and testing, including BTL detection, and unbiased estimation of both penetrance and BTL location.

SIMON TAVARÉ, Department of Mathematics, University of Southern California, Los Angeles, California 90089-1113, USA
The genealogy of branching processes and the reconstruction of tumor histories

Genealogical approaches have found many applications in population genetics, where the coalescent has played a crucial role in understanding molecular variability in natural populations. Related methods are also proving useful for the study of mitotic division processes. We have been using a quantitative analysis of microsatellite mutations in colorectal tumors to infer their histories, and to shed light on the likely nature of the adenoma-carcinoma sequence. The main tool we use is an algorithm for generating the genealogy of a sample of cells evolving according to a discrete time (possibly non-Markovian, time-inhomogeneous) branching process. A number of applications, including estimation of the age of a tumor and generalizations to multitype processes, will be discussed.

E. A. THOMPSON, University of Washington, Washington, USA
Conditional genome sharing from dense marker maps

With increasing marker data availability and ever-improving genetic maps, localization of the genes contributing to complex traits remains a hard problem. Methods for estimating gene locations are sensitive to trait model assumptions, particularly when multiple markers are analyzed jointly. Robust methods lack power for linkage detection, and localization can be problematic. Data on multiple relatives can provide more power, but valid analysis of data on multiple relatives jointly at multiple marker loci raises severe computational issues.

Markov chain Monte Carlo (MCMC) methods provide realizations of gene identity by descent among pedigree members, conditional on data at multiple marker loci, in situations in which exact computation is infeasible. This estimated gene sharing permits detection and localization of genes contributing to a trait, determination of the pedigrees a trait allele, and inference of gene carriers.

Orders, Lattices and Universal Algebra / Ordres, treillis et algèbre universelle

(L. Haddad, B. Larose and I. Rosenberg, Organizers)

STANLEY BURRIS, University of Waterloo
Density in abstract number systems

The finite algebras in varieties with the unique factorization property provide excellent examples of abstract number systems where one can invoke the Dirichlet Series = Euler Product to analyze the density of partition sets, and this in turn can be used to study the probability that a first-order sentence hold in the finite models of the variety.

ISIDORE FLEISHER, CRM, Montreal, Quebec
Functional representation of pre-iterative/combinatory formalism

GEORGE GRATZER, Department of Mathematics, University of Manitoba, Winnipeg Manitoba R3T 2N2
On the endomorphism monoids of (uniquely) complemented lattices

In 1970, the authors proved the following result:

Theorem 1 *Every monoid $\mathcal{M}$ can be represented as the $\{0,1\}$-endomorphism monoid of a suitable bounded lattice L.*

Now we can prove the following two results:

Theorem 2 *Every monoid $\mathcal{M}$ can be represented as the $\{0,1\}$-endomorphism monoid of a suitable complemented lattice L. Moreover, if $\mathcal{M}$ is finite, then L can be chosen as a finite complemented lattice.*

Theorem 3 *Every monoid $\mathcal{M}$ can be represented as the $\{0,1\}$-endomorphism monoid of a suitable uniquely complemented lattice L.*

Theorem 2 solves Problem VI.24 of G. Grätzer's *General Lattice Theory*, (1978).

Recall that uniquely complemented lattices are very difficult to construct. R. P. Dilworth in 1945 solved a long standing conjecture of lattice theory by proving that not every uniquely complemented lattice is distributive (Boolean). He proved this by examining free lattices with a "free" complement operation. Free algebras have very special $\{0,1\}$-endomorphism monoids since every map of the generators can be extended to a $\{0,1\}$-endomorphism. So we were quite surprised that Theorem 1 could be sharpened to Theorem 3.

The proof of these results relies on several results in the literature, due to C. C. Chen and G. Grätzer 1969, H. Lakser 1972, M. E. Adams and J. Sichler 1977, V. Koubek and J. Sichler 1984.

JENNIFER HYNDMAN, Department of Mathematics and Computer Science, University of British Columbia, British Columbia
Strong duality of finite algebras that generate the same quasivariety

Dualisability of a quasivariety is independent of the finite generating algebra. In this talk we discuss the result that strong dualizability of a quasivariety is independent of the finite generating algebra.

HANS-KARL KEISER, Wein, Austria
To be announced

HAJIME MACHIDA, Hitsotsubasji University, Kunitachi, Tokyo 186-8601, Japan
Hyperclones on the two-element setzo

Recently, I. G. Rosenberg initiated the study of hyperoperations and hyperclones. For a set A, a hyperoperation on A is a mapping from $A \times \cdots \times A$ to the set of non-empty subsets of A and a hyperclone on A is a composition-closed set of hyperoperations. Here we study some basic properties of hyperoperations and hyperclones. In particular, we show that the lattice of hyperclones on the two-element set $\{0, 1\}$ has the cardinality of continuum. This answers affirmatively to Rosenberg's problem posed in 1998.

ALIOUNE NGOM, Lakehead University
Set-valued logic algebras

GRANT POGOSYAN, Japan
Irreducible clones

BOB QUACKENBUSH, University of Manitoba
Duality and nonduality theorems for finite groups

LUIGI SANTOCANALE, Département de mathématiques, Université du Québec à Montréal, Montréal, Québec H3C 3P8
Free μ-lattices[1]

If P is a partially ordered set and ϕ is an order preserving function from P to P, the least prefix-point of ϕ is an element μ of P such that $\phi(\mu) \leq \mu$ and such that if $\phi(p) \leq p$, then $\mu \leq p$. The greatest postfix-point is defined dually.

A lattice is a μ-lattice if every unary polynomial has a least prefix-point and a greatest postfix-point. For a unary polynomial we mean a derived operator evaluated in all but one variables; operators are derived from the basic ones of lattice theory by substitution and by "taking fix-points". A category of μ-lattices is defined and it turns out to be a quasivariety.

For a given partially ordered set P, we describe a μ-lattice J_P by means of games: we define a class $J(P)$ whise elements are games and a preorder on it by saying that, for $G, H \in J(P)$, $G \leq H$ if and only if a specified player has a winning strategy in a compound game $[G, H]$. This relation is shown to be decidable if the order of P is decidable.

By showing that J_P is free over P we give a solution to the word problem for the theory of μ-lattices.

CLAUDE TARDIF, Regina
Projectivity and product colourings

WILLIAM TROTTER, Arizona State University, Arizona, USA
To be announced

[1]The research presented in this talk has been developed as part of doctoral studies at the Université du Québec à Montréal under the supervision of Prof. André Joyal.

SHELLY WISMATH, Department of Mathematics and Computer Science, University of Lethbridge, Lethbridge, Alberta T1K 3M4
Hyperidentities for varieties of star bands

A hyperidentity of a variety of algebras is an identity which holds in an additional stronger way. This talk will give an introduction and short overview of the study of hyperidentities, then present some joint work with J. Koppitz on the hyperidentities satisfied by the varieties of star-bands. This is based on an equational description of the lattice of all varieties of star-bands given by Adair and by Petrich.

LASZLO ZADORI, U. Szeged
Finite posets with symmetric idempotent operations

An n-ary operation f is totally symmetric if it obeys the identity $f(x_1, \ldots, x_n) = f(y_1, \ldots, y_n)$ for all sets of variables such that $\{x_1, \ldots, x_n\} = \{y_1, \ldots, y_n\}$. A characterization of finite posets admitting an n-ary idempotent totally symmetric operation for all n is given. The characterization is expressed in terms of zigzags, special objects assigned to the poset. Related problems concerning idempotent Malcev conditions for order primal algebras are mentioned in the talk.

Teaching of Linear Algebra / L'enseignement de l'algèbre lin'eaire

(J. Hillel, J. Klasa and V. Hussin, Organizers)

JOHN AUER, Department of Mathematics, Brock University, St. Catharines, Ontario L2S 3A1
Ten years of teaching linear algebra using Maple V at Brock University

Since the late 1980s, Brock University has offered a freshman linear algebra, one-semester course for a wide audience, using the Maple V CAS. This led to the publication of my first linear algebra text (Prentice Hall, 1990) containing Maple V supplementary material for use in solving linear algebra problems in a weekly, compulsory one hour laboratory-tutorial. A second, fully integrated text entitled "Essentials of Linear Alegebra Using Maple V" (1999) has been published and is used as the text and laboratory manual for this course.

A CAS like Maple V or Mathematica is an ideal environment for solving problems and exploring concepts in linear algebra. My talk will discuss our course content and the integration of Maple V, concentrating on the following features provided by CASs:

(i) The facilitation of focusing on concepts and methods, without the distraction of numerical mistakes in lengthy calculations. A case in point is the learning of row reduction of matrices, where CASs contain individual commands for the three elementary row operations. Of course, CASs also have "canned" commands to produce the final result (*e.g.*, a row echelon matrix), but these should be avoided, except as noted in (iii) below.

(ii) The exploration of theoretical results using CASs' symbolic manipulation capabilities. One simple example is provided by the exploration of the structure of the formulas for the determinant of a matrix W traditionally bewildering for many students.

(iii) The checking of numerical and symbolic calculations. I think this is very beneficial in building

students' self-confidence.

JEFF BOATS, Department of Mathematics and Computer Science, University of Michigan, Detroit, Michigan 48219-0900, USA
On using computer tutorials to tailor linear algebra for secondary teachers

An integral part of a mathematics student's education involves developing an intuition about underlying structures and relationships. Even more so for the future mathematics educator, who will need to illustrate these insights. The courses in a mathematics education program should be crafted to develop such intuition.

The linear algebra student, in particular, must develop the ability to visualize the properties of vectors and vector spaces. But while an abstract understanding of vector spaces benefits the mathematician, the secondary mathematics teacher requires a more concrete visualization. The goal, then, is to provide resources with which the future math educator may improve his visualization.

Properly-designed computer tutorials can be used to decompose the standard problems of linear algebra into simpler, more digestible forms. These tutorials generally involve exercises in construction, and take full advantage of the computer's speed and efficiency, avoiding the pitfall of computer-empowerment.

This talk briefly describes the motivation of computer tutorials, explains how they differ from the computer exercises of most textbooks, and provides an example or two.

BILL BYERS, Department of Mathematics and Statistics, Concordia University, Montreal, Quebec H4B 1R6
Working with ambiguity in linear algebra

We usually think of ambiguity in mathematics as something that must be avoided. Nevertheless there are certain "positive ambiguities" which arise in the teaching of Linear Algebra: concepts that must be thought of in a flexible, multi-dimensional way. Take, for example, the many ways in which we think of a matrix: as an array of numbers, a linear transformation, a set of row vectors, etc. To learn Linear Algebra means in part to be able to move flexibly from one of these conceptions of a matrix to another.

It is precisely the ambiguous nature of Abstract Linear Algebra that makes the transition from computational Matrix Algebra so difficult for the student. How then do we present ambiguous concepts to the student? What does it mean for a student to learn a concept that is deep and multi-faceted? These are difficult questions and I have no easy answers. Rather I intend to discuss my experience in looking at the teaching of Linear Algebra from this point of view.

BILL CASSELMAN, Department of Mathematics, University of British Columbia, Vancouver, British Columbia
Linear algebra with a Java-based programmable calculator

For a course in linear algebra taught in the Spring of 1999 to engineering students at UBC, I developed labs and coursework using a a Java-based programmable calculator, somewhat reminiscent of one of the high end HP graphing calculators. The calculator uses its own RPN language. It runs inside an Internet browser. It includes a fairly complete programming environment and a pop-up graphics window. Documentation, the calculator itself, and related course material can all be seen at sites referred to on the page

http://gamba.math.ubc.ca/coursedoc/math152/index.html

It was an interesting experiment. Success with students depended very much on how closely an instructor integrated the calculator into general course material, but the potential seems to be

enormous. Among other things, it is very simple to extend the calculator's capabilities with a very small amount of Java program coding.

DANIEL NORMAN, Department of Mathematics and Statistics, Queen's University, Kingston, Ontario K7L 3N6
Teaching linear algebra independence via unique representation

The important role of linear independence in introductory linear algebra is to guarantee the uniqueness of coordinates defined with respect to a basis. It is suggested that students might grasp the essential idea better if the idea of unique representation was stressed first, instead of either proceeding straight to a formal definition or first discussing examples of geometrical dependence of vectors.

ASUMAN OKTAC, Department of Mathematics and Statistics Concordia University, Montreal, Quebec H4B 1R6
Linear algebra: Is possible at a distance

Introductory linear algebra was taught as a distance education course as part of a masters in education program for inservice teachers. It relied on satelite classes, e-mail discussions and chat sessions to establish communication between the members of the course. Emphasis in this talk will be on the types of interaction that took place among different virtual student groups and the mathematical content of these exchanges. We will also concentrate on the differences between a regular setting and this medium, and their consequences for teaching and learning.

MORRIS ORZECH, Department of Mathematics and Statistics, Queen's University, Kingston, Ontario K7L 3N6
Linear algebra and rigour—mixed messages as an opportunity

I teach linear algebra to students just out of high school. My class consists of prospective majors in mathematics and theoretical physics, and other honours students who like mathematics and want strong preparation for its use and study. Exposure to mathematical rigour is a fitting ingredient for this course, but most of us would recognize the need to balance it against constraints such as time, student preparedness and aesthetics. What do students make of this "balancing act"? Most apprehend it as a mixed message that engenders confusion about what is expected of them. Making our expectations explicit ("Here is a list of proofs you should know.") is sensible, but by itself misses an opportunity to confront students with practical and intellectual aspects of proof strategy and proof culture. Finding problems and activities to exploit this opportunity is challenging, but my experience indicates it can also be a good learning experience for student and teacher.

DAVID POOLE, Department of Mathematics, Trent University, Peterborough, Ontario K9J 7B8
Does linear algebra need to be "reformed"?

In this talk, I will offer some personal reflections on linear algebra textbooks and curricula, past and present. My comments will be informed by recommendations made by the Linear Algebra Curriculum Study Group (*cf.* College Mathematics Journal, January 1993) as well as my own teaching experiences. I hope to be sufficiently provocative to generate some lively, but friendly, discussion.

ANNA SIERPINSKA, Department of Mathematics and Statistics, Concordia University, Montreal, Quebec H4B 1R6
Practical, theoretical, synthetic and analytic modes of thinking in linear algebra

The talk will focus on certain aspects of students' reasoning in linear algebra that may be responsible for their perceived difficulties in the domain. It will be argued that students tend to think in practical rather than theoretical ways and that, among the theoretical ways of thinking, the structural mode is the least accessible. The different modes of thinking will be described in detail, and the ways in which some of them may function as obstacles to students' understanding will be illustrated by examples of actual student behavior.

The examples will be drawn from a series of research projects conducted at Concordia University in the years 1993–1999 by myself and Joel Hillel in collaboration with, at various stages, Tommy Dreyfus and Jana Trgalova. In the years 1993–96 our research focused on patterns of interaction between individual students, their tutors and texts and their influence on the students' learning of linear algebra. The mathematical content in these observations was not designed by the researchers: it was the mathematical content of the existing linear algebra undergraduate texts. On the other hand, the design of an entry into linear algebra was at the core of the projects in the years 1996–1999. The design was based on a geometric model of the two-dimensional vector space within the dynamic geometry environment of the Cabri-geometry II software. The design had undergone a cycle of three experimentations followed by amendments to the design on the basis of the students' reactions.

GILBERT STRANG, Department of Mathematics, MIT, Cambridge, Massachusetts 02139, USA

Partly random graphs and small world networks

It is almost true that any two people in Canada are connected by less than six steps from one friend to another. What are models for large graphs with such small diameters?

Watts and Strogatz observed (in Nature, June 1998) that a few random edges in a graph could quickly reduce its diameter (longest distance between two nodes). We try to analyze this. We also study a related model, which starts with n edges around a cycle (large diameter) and adds n edges around a second (but now random) cycle. The average distance between pairs becomes nearly $A \log n + B$. The eigenvalues of the adjacency matrix are surprisingly close to an arithmetic progression; for each cycle they would be cosines, the sum changes everything.

We will discuss some of the analysis (with Alan Edelman and Henrik Eriksson at MIT) and also some applications. We also report on the surprising eigenvalue distribution for trees (large and growing) found with Li He and Xiangwei Liu. In this case we found eigenvalues with very high multiplicity.

Contributed Papers / Communications libres

(L. Haddad, Organizer)

PAUL ARMINJON, Universite de Montreal, Centre de Recherches Mathematiques, Montreal, Quebec H3C 3J7

Numerical computation of 3*-D flows with a non-oscillatory central scheme on unstructured tetrahedral grids*

We present a 3-dimensional Lax-Friedrichs-type central finite volume method for hyperbolic equations and systems on unstructured tetrahedral grids.The resolution of the Riemann problems at the cell interfaces is avoided thanks to the use of the staggered Lax-Friedrichs scheme. Piecewise linear cell interpolants with van Leer-type limiting techniques to estimate the gradients lead to a quasi-second order accurate non-oscillatory resolution. Numerical results for a linear advection problem

and for the 3-D Euler equations (supersonic flow through a channel with a bump) are presented.

LYUDMILA BANTSUR, Ternopil State Pedagogical University, Ternopil, Ukraine
The method of stability analysis of nonlinear systems using Liapunov functions in the form of norms is proposed

The Jacobian matrix set of the system on some positively invariant set is considered. It is show that the autonomous global Liapunov function in the form of norm exist if and only if the Jacobian matrice are simultaneously dissipative, *i.e.* such a norm exit that all the matrices considered generate contractive semigroups in this norm. Some conditions for simultaneous dissipativity are discussed. The conditions obtained allow to judge the stability "in the large". The conditions for the existence of the Liapunov norm for the perturbed autonomous system with positively homogeneous right-hand side are obtained.

NATALIYA BANTSUR, Institute of Mathematics of the NAS of Ukraine, Kyiv 252601 Ukraine
Existence of T-periodic solutions of nonlinear scalar differential equations with maxima

Problems of the existence of T-periodic solutions of nonlinear scalar superlinear differential equations with maxima of the form

$$x'(t) = f\big(x(t)\big) + g\big(\max_{u\in[t-h;t]} x(u)\big) + p(t) \quad t, x \in R$$

is investigated. Part of these results is joint work with E. Trofimchuk.

References

1. N. R. Bantsur and E. P. Trofimchuk, *About existence of T-periodic solutions of nonlinear scalar differential equations with maxima*. Nonlinear oscillation **1**(1998), 1–5 (in Ukraine).
2. _____, *Existence and stability of the periodic and almost periodic solutions of quasilinear systems with maxima*. Ukrain. Mat. Zh. **1**(1998), 747–753 (in Ukraine).

BORIS DEKSTER, Mount Allison University, Sackville, New Brunswick E4L 1E6
Each convex body in $E3$ symmetric about a plane can be illuminated by 8 directions

Let C be a convex body in E^d, $d \geq 2$. Let x be a point on ∂C and v be a direction (non-zero vector). Consider the axis l having the direction v and passing through x. The direction v is said to *illuminate* x if l contains a point $y \in \operatorname{int} C$ which succeeds x. If each point of a part of ∂C is illuminated by at least one of a few directions, the body C is said to be illuminated by these directions. We prove the following.

Theorem *Each convex body in E^3 symmetric about a plane can be illuminated by 8 directions.*

For polyhedral bodies, this result was established by K. Bezdek in 1991. Our method however is quite different. Both results are partial proofs of the Hadwiger Conjecture according to which each convex body in E^d can be illuminated by 2^d directions.

PAUL GAUTHIER, Universit/'e de Montréal, Montréal, Québec H3C 3J7
The Cauchy and Green theorems pour connexite arbitraire

The fundamental theorem of complex analysis is the Cauchy theorem which states that if D is a bounded domain of the complex plane with rectifiable boundary, consisting of finitely many disjoint Jordan curves, then

$$\int_{\partial D} f(z)\, dz = 0,$$

for each function f holomorphic in D and continuous on its boundary. We show that the word 'finitely' is superfluous. Nous donnons aussi une généralisation analogue du théorème de Green.

MYKOLA HLADIY, Ternopil State Pedagogical University, Ternopil, Ukraine
Generalized Green's operator of boundary value problem with impulse action

We find necessary and suficient conditions for solvability of nonhomogeneous linear boundary value problems for systems of ordinary diferential equations with impulsive force in general case where the number of bounadry value conditions is not equal to the order of the differential systems.

We construct a generalized Green's operator for boundary value problems, not every solutions of which can be extendnfrom left end point to the right end point of the interval where the solution is defined.

References

A. M. Samoilenco and N. A. Perestyuk, *Differential equations with impulsive action.* Kyiv: Vyscha shkola, 1987, 287 pages (in Russian).

O. KIHEL ET C. LEVESGUE, Université Laval, Laval, Québec G1K 7P4
Sur un problème de Diophante

Soit n un entier. On dit qu'un ensemble de m entiers positifs $\{x_1, x_2, \ldots, x_m\}$ possède la propriété P_n (appelée aussi *propriété de Diophante*) si pour tout i, j avec $i \neq j$, $x_i\, x_j + n$ est un carré parfait. Le problème de trouver de tels ensembles remonte à Diophante. Fermat a remarqué que $\{1, 3, 8, 120\}$ possède la propriété P_1. Baker et Davenport ont montré que cet ensemble *ne peut pas s'étendre,* en ce sens qu'il n'existe aucun entier a tel que $\{1, 3, 8, 120, a\}$ possède la propriété P_1. On présentera de nouvelles familles ayant la propriété P_n pour certains n, et on montrera que certaines familles ne peuvent pas s'étendre.

O. KIHEL ET C. LEVESGUE, Université Laval, Laval, Québec G1K 7P4
Sur les sommes de puissances consécutives

Il est facile de voir que l'équation $n^2 + (n+1)^2 = m^2$ admet une infinité de solutions entières et que l'équation $x^3 + (x+1)^3 + (x+2)^3 = y^3$ est vérifiée seulement pour $x = 3$ et $y = 6$. Qu'en est-il de l'équation

$$x^n + (x+1)^n + \cdots + (x+k)^n = y^m?$$

Dans cet exposé, nous étudions ce genre d'équations. Nous résolvons le problème lorsque $x = 1$, $n = 1$, 3, 4 et nous étudions la question pour d'autres valeurs de n et m.

PETER LANCASTER, Department of Mathematics and Statistics, University of Calgary
Calgary, Alberta, T2N 1N4
Numerical ranges of selfadjoint quadratic matrix polynomials

Consider monic matrix polynomials $P(\lambda) = I\lambda^2 + A_1\lambda + A_0$, where A_0 and A_1 are $n \times n$ Hermitian matrices and λ is a complex variable. The numerical range of such a polynomial is

$$W(P) = \{\lambda \in \mathbb{C} : x^* P(\lambda) x = 0, \text{ for some nonzero } x \in \mathbb{C}^n\}$$

and it always contains the spectrum of P, $\sigma(P)$, *i.e.* the set of zeros of $\det P(\lambda)$. Properties of the numerical range are to be reviewed, taking advantage of the close connection between $W(P)$ and the classical numerical range (field of values) of the (general) complex matrix $A := A_0 + iA_1$.

The study of eigenvalues and non-differentiable points on the boundary is of special interest. We consider also the problem of the numerical determination of $W(P)$ and illustrate with examples generated with the help of "matlab". Comments are made on extension of the theory to more general polynomials $P(\lambda)$, related factorization results, and the connection with the recently introduced "quadratic numerical range" when applied to the companion matrix of P.

This is a report on joint work with Panayotos Psarrakos.

JUN LI, Département de mathématiques et de statistique, Université de Montréal, Montréal, Québec H3C 3H7

Asymptotic behavior of a linear vector recurrence

We describe the asymptotic behavior of some linear vector recurrences of the form $v_n = Av_{n-1} + b$. This behavior depends mainly on the dominant Jordan matrice associated with A. The analysis will be done by dealing with two particular cases: 1 is not an eigenvalue of A and 1 is the only eigenvalue of A. In the general case, the asymptotic behavior will be obtained by decomposing the vector space into the direct sum of two invariant vector subspaces.

WEI-JIU LIU, Dalhousie University, Halifax, Nova Scotia B3H 3J5

Adaptive control of Burgers' equation with unknown viscosity

In this talk, we propose a fortified boundary control law and an adaptation law for Burgers' equation with unknown viscosity, where no a priori knowledge of a lower bound on viscosity is needed. This control law is decentralized, *i.e.*, implementable without the need for central computer and wiring. Using the Lyapunov method, we prove that the closed-loop system, including the parameter estimator as a dynamic component, is globally H^1 stable and well posed. Furthermore, we show that the state of the system is regulated to zero by developing an alternative to Barbalat's Lemma which can not be used in the present situation.

F. PERRON, ?

To be announced

DIETER RUOFF, University of Regina, Regina, Saskatchewan

Proportionality in the non-Euclidean plane

The Euclidean proportionality theorems involving an angle that is intersected by a pair of parallel lines do not extend to the hyperbolic plane; on the the one hand the uniqueness of the parallel line and on the other a meaningful concept of similarity are missing. In fact, the proportionality theorems also fail when parallelism is interpreted in the narrower sense of being boundary parallel.

What is well-known is the theorem that a line which bisects two sides of a triangle is hyperparallel to the third side. Taking this as a rudimentary proportionality theorem one can ask whether the pair of lines that trisects two sides of a triangle would still be hyperparallel to the third, *etc.* As will be shown the proof that this is so is not straightforward but requires some interesting lemmas concerning the hyperbolic plane.

KONSTANTIN RYBNIKOV, Department of Mathematics and Statistics, Queen's University Kingston, Ontario K7L 3N6

Loss of tension in an infinite membrane with holes distributed by a Poisson law

If one randomly punches holes in an infinite tensed membrane, when does the tension cease to exist? This problem was introduced by R. Connelly in connection with applications of rigidity theory

to natural sciences. We outline a mathematical theory of tension based on graph rigidity theory and show that if the "centers" of the holes are distributed in $\mathbb{R}^2$ according to a Poisson law with parameter $\lambda > 0$, and the distribution of the shapes of the holes is independent of the distribution of their centers, the tension vanishes on all of $\mathbb{R}^2$ for any value of λ. In fact, this result follows from a more general theorem on the behavior of iterative convex hulls of connected subsets of $\mathbb{R}^d$, where the initial configuration of subsets is distributed according to a Poisson law, and the shapes of the elements of the original configuration are independent of this Poisson distribution. For the latter problem we establish the existence of a critical threshold in terms of the number of iterative convex hull operations required for covering all of $\mathbb{R}^d$. The processes described in the paper are somewhat related to bootstrap and rigidity percolation models. This is a M. V. Menshikov and S. E. Volkov.

CRISTINA STOICA, University of Victoria

The relative two-body problem in quasi-homogeneous potentials fields

The relative two-body problem in a so-called quasi-homogeneous potential field (*i.e.* the interaction between the particles is generated by a sum of the form $W = U + V$, with U and V homogeneous functions in $1/r$, r being the distance between the particles) is presented.

Using the McGehee type diffeomorphisms and a suitable reparametrization of time, the initial second order differential system is transformed to an analytic system. The flow is constrained to be on the energy manifold, here a class $C - 1$ relation.

The submanifold $r = 0$, called the *collision manifold*, is invariant under the flow and the angular momentum conservation isolates the angular velocity component.

We will offer a full description of the orbits on and about the collision manifold, pointing out for each different case with respect to the degrees of U and V the Lebesgue measure of the set of initial conditions leading to collision.

JAMES WATMOUGH, University of Victoria, Victoria, British Columbia V8W 3P4

A simple SIS *epidemic model with a backward bifurcation*

Classical disease transmission models with constant contact rates typically have only a single stable equilibrium. There is a threshold level of the reproduction number below which the disease dies out and above which the disease approaches an endemic level.

In this talk I will formulate an nonlinear Volterra integral equation to model the dynamics of a simple disease. The model always possesses a disease free equilibrium (zero infectives) which loses stability as a parameter (the basic reproduction number) is increased through a threshold. In classical models, this bifurcation is always super critical. However, with the introduction of a non-constant contact rate, the bifurcation may be sub-critical, implying the existence of multiple equilibrium and hysterisis.

Let $I(t)$ and $S(t) = 1 - I(t)$ be the fraction of the population in each of the two disjoint classes, infective and susceptible, at time $t \geq 0$. Susceptible individuals become infective at a rate $\lambda\big(I(t)\big)I(t)S(t)$. The integral equation studied has the form

$$I(t) = I_o(t) + \int_0^t \lambda\big(I(u)\big)I(u)\big(1 - I(u)\big)P(t-u)e^{-b(t-u)}\,du, \tag{1}$$

where $S(t)$ has been replaced by $1-I(t)$. Briefly, $P(t-u)e^{-b(t-u)}$ is the probability that an individual infected at time u is still infectious at time t. The birth and death rate of individuals are both equal to the parameter b. The integral sums the individuals that entered the infective class at time $u \geq 0$ and have remained infective through to time t.

We determine the existence and stability of equilibria of Equation (1) as a function of the reproduction number

$$R_o = \lambda(0)\int_0^\infty P(u)e^{-bu}\,du, \tag{2}$$

which is the expected number of infectives produced by a single infective during its lifetime. There are two thresholds of the reproduction number, $R_o^m \geq R_o^c > 0$, such that the disease free equilibrium is the only equilibrium solution and is globally asymptotically stable for $R_o < R_o^c$, and there is a single, globally asymptotically stable endemic (positive) equilibrium for $R_o > R_o^m$. Sufficient conditions for the existence of multiple stable equilibria are given by the following result.

Theorem 1 *For the model of Equation (1), with the assumptions of the previous section and* $\lambda(0) > 0$*, there is a transcritical bifurcation at* $R_o = 1$*,* $\bar{I} = 0$*. This bifurcation is in the forward direction if* $\lambda'(0) < \lambda(0)$ *and in the backward direction if* $\lambda'(0) > \lambda(0)$*. Further, if the model has a backward bifurcation, then* $R_o^c < 1$ *and there are multiple stable equilibria for* $R_o^c < R_o < 1$*.*

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