
Symplectic Geometry with constraints from Algebraic Topology
Géométrie symplectique avec contraintes issues de la topologie algébrique

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A construction of polyptych lattices via a pair of Gorenstein PL cones

This is joint work with Christopher Manon and Laura Escobar. The theory of polyptych lattices is motivated by a desire to simultaneously generalize phenomena in toric, cluster, and tropical geometry, among others. In particular, the theory builds a combinatorics-geometry dictionary reminiscent of that in toric geometry. The theory builds the notion of mutations directly into the definitions, and gives geometric interpretations to these mutations. In particular, in some cases, a polyptych-lattice ("PL") polytope encodes a compactification $\mathcal{X}$ of an affine variety $U = \text{Spec}(\mathcal{A})$ whose coordinate ring $\mathcal{A}$ can be equipped with a valuation to a canonical semialgebra associated to the polyptych lattice. This is analogous to the known phenomenon that a toric variety may be viewed as a compactification of a complex torus $(\mathbb{C}^*)^n$ by toric divisors, and the complex torus can be viewed as Spec of the Laurent polynomial ring, which can in turn be equipped with the valuation that takes a Laurent polynomial to its Newton polytope. Moreover, aspects of the geometry of the compactification $\mathcal{X}$ can be understood combinatorially; for instance, in some cases, the resulting compactifications are arithmetically Cohen-Macaulay, and have finitely generated class group and finitely generated Cox rings. The theory also naturally encodes multiple toric degenerations of the compactification $\mathcal{X}$ to toric varieties whose moment map images are related by mutations. In this talk, I recall the setup of this theory, and sketch a new construction of polyptych lattices coming from a pair of Gorenstein PL cones.

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What is Symplectic Geometry? The Weinstein "category" and the dual dg category

In Weinstein's symplectic "category", objects are symplectic manifolds and morphisms are canonical relations, which may not be composable; it is therefore only morally a category. We construct a true differential graded category which is morally dual to Weinstein's "category". The objects in this category are prequantum systems, associated to integral symplectic manifolds. The morphisms are a complex of differential forms twisted by a prequantum line bundle. We also consider the cohomology category, which turns out to be an (ordinary) linear category. In each case, Weinstein's morphisms are associated with currents dual to the forms. For the differential graded category, these are isotropic, or if preferred Lagrangian, currents, twisted by a section of the prequantum line bundle; in the case of cohomology, these currents are supported on *integral* Lagrangian submanifolds, equipped with a global covariant constant section of the prequantum line bundle. We then apply our methods to quantization, and show that for Kahler manifolds, a quantization in this category is related to holomorphic quantization. Along the way we show that in the case of prequantum systems, the Lefschetz action of $\mathfrak{sl}(2, R)$ on differential forms, studied by Brylinski, Mathieu, Guillemin, and Tseng-Yau in the symplectic case, is promoted to an action of the superalgebra $\mathfrak{osp}(1|2)$ on twisted differential forms, with $\mathfrak{sl}(2, R)$ as the even subalgebra. A first application of this superalgebra action is the vanishing theorem which shows the cohomology category is supported in middle dimension.

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