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What is Symplectic Geometry? The Weinstein "category" and the dual dg category

In Weinstein's symplectic "category", objects are symplectic manifolds and morphisms are canonical relations, which may not be composable; it is therefore only morally a category. We construct a true differential graded category which is morally dual to Weinstein's "category". The objects in this category are prequantum systems, associated to integral symplectic manifolds. The morphisms are a complex of differential forms twisted by a prequantum line bundle. We also consider the cohomology category, which turns out to be an (ordinary) linear category. In each case, Weinstein's morphisms are associated with currents dual to the forms. For the differential graded category, these are isotropic, or if preferred Lagrangian, currents, twisted by a section of the prequantum line bundle; in the case of cohomology, these currents are supported on *integral* Lagrangian submanifolds, equipped with a global covariant constant section of the prequantum line bundle. We then apply our methods to quantization, and show that for Kahler manifolds, a quantization in this category is related to holomorphic quantization. Along the way we show that in the case of prequantum systems, the Lefschetz action of $\mathfrak{sl}(2, R)$ on differential forms, studied by Brylinski, Mathieu, Guillemin, and Tseng-Yau in the symplectic case, is promoted to an action of the superalgebra $\mathfrak{osp}(1|2)$ on twisted differential forms, with $\mathfrak{sl}(2, R)$ as the even subalgebra. A first application of this superalgebra action is the vanishing theorem which shows the cohomology category is supported in middle dimension.