Spectral Geometry
Géométrie spectrale
(Org: **Maxime Fortier Bourque** (Université de Montréal), **Alexandre Girouard** (Université Laval) and/et **Lawford Hatcher** (Université de Montréal))

SAMUEL AUDET BEAUMONT, Université Laval

THOMAS BECK, Fordham University
Nodal domain estimates for dumbbell domains

In this talk, we will discuss the nodal geometry of Neumann eigenfunctions of dumbbell domains, as the width of the thin neck joining the two sub-domains shrinks to zero. We will see that, generically, this forces the number of nodal domains to grow with the eigenvalue, and that in particular the third eigenfunction is Courant sharp (attains equality in the Courant nodal domain theorem) for sufficiently thin necks. In a symmetric case, we will also describe a criterion guaranteeing the existence of pairs of eigenfunctions that are Courant sharp. The proofs of these results require the leading order, non-zero, asymptotics of these eigenfunctions in each part of the dumbbell, which is established via a matched asymptotic analysis with logarithmic terms.

This is based on joint work with Yaiza Canzani, Andrew Lyons, and Jeremy Marzuola.

JARED BRONSKI, University of Illinois Urbana-Champaign

DMITRY FAIFMAN, Université de Montréal

JOSEF GREILHUBER, Massachusetts Institute of Technology

THOMAS HÉLIÈRE, Université Laval

DMITRY JAKOBSON, McGill
Small eigenvalues of degenerating Riemann surfaces

It is known that the small eigenvalues of the Laplacian of a Riemann surface close to the boundary of the modular space can be well approximated by the eigenvalues of the discrete Laplacian on a certain graph coming from the pair of pants decomposition of the surface. We provide a complete description of the sets of eigenvalues of the weighted graph Laplacian for all graphs on four vertices that correspond to a valid pair of pants decomposition of a surface of genus 3.

HANNA KIM, University of North Carolina

RICHARD LAUGESEN, University of Illinois Urbana–Champaign
Rayleigh's conjecture for vibrating plates: simpler planar proof and nonlinear extensions

Which shape of clamped plate achieves the lowest fundamental tone? Rayleigh conjectured that among planar plates of fixed area, the disk minimizes the first eigenvalue of the bilaplacian. This fourth-order analogue of the Faber–Krahn theorem is substantially harder than the membrane problem, and was not proved in two dimensions until 1995, by Nadirashvili.

I will present a new and simpler proof, based on a paired comparison principle for Poisson equations with sign-changing data. The method extends naturally to a whole family of nonlinear clamped-plate problems: the disk minimizes the nonlinear Rayleigh quotient

$$\frac{\|\Delta u\|_{L^q(\Omega)}}{\|u\|_{L^p(\Omega)}}, \qquad u = \partial_\nu u = 0 \quad \text{on } \partial\Omega,$$

for every $1 \le p, q \le \infty$. Thus the original linear spectral conjecture opens the door to a much larger nonlinear universe.
(Joint work with Dorin Bucur.)

NILIMA NIGAM, Simon Fraser University

KSHITIJ PATIL, Simon Fraser University

FRÉDÉRIC ROCHON, Université du Québec à Montréal

EGOR SHELUKHIN, Université de Montréal

DAVID SHER, DePaul University
Heat trace and Riesz means with a large Robin parameter

We consider the Laplacian with Robin boundary conditions on a smooth Riemannian manifold with boundary and investigate the behavior of various spectral quantities as the Robin parameter approaches infinity. Although the spectrum, heat trace, and Riesz means converge pointwise to their Dirichlet analogues, the $asymptotics$ of these quantities as the spectral parameter approaches infinity, or as time approaches zero, do not. We investigate this phenomenon and explain the divergent behavior. Our main result is a two-term asymptotic expansion for the Robin heat trace that depends jointly on the time variable and the Robin parameter, with a remainder that satisfies a uniform bound independent of the Robin parameter. We apply this result to obtain a similar two-term asymptotic expansion for Riesz means.

ROMAIN SPECIEL, Stanford University

ALINA STANCU, Concordia University

JOHN TOTH, McGill

Non-concentration estimates for Laplace eigenfunctions on compact C^∞ manifolds with boundary

Let Ω be an n-dimensional compact Riemannian manifold with C^∞ boundary, and consider L^2-normalized eigenfunctions $-\Delta\phi_\lambda = \lambda^2\phi_\lambda$ with Dirichlet or Neumann boundary conditions. In this talk, I will describe reccent joint work with H. Christianson (UNC) on the extension of interior nonconcentration bounds up to the boundary. Specifically, using purely stationary local methods, we prove that for such Ω it follows that for *any* $x_0 \in \overline{\Omega}$ (including boundary points) and for all $\mu \geq C_\Omega h$ with sufficiently large constant $C_\Omega > 0$,

$$\|\phi_\lambda\|^2_{B(x_0,\mu)\cap\Omega} = O(\mu). \tag{1}$$

We also extend a result of C. Sogge to manifolds with smooth boundary and show that when $n \geq 3$,

$$\|\phi_\lambda\|_{L^\infty(\Omega)} \leq C\lambda^{\frac{n}{2}} \cdot \Big(\sup_{x\in\Omega} \|\phi_\lambda\|_{L^2(B(x,\lambda^{-1})\cap\Omega)}\Big). \tag{2}$$

The sharp sup bounds $\|\phi_\lambda\|_{L^\infty(\Omega)} = O(\lambda^{\frac{n-1}{2}})$ for Dirichlet or Neumann eigenfunctions proved by D. Grieser are then an immediate consequence of these bounds.

DENIS VINOKUROV, Université de Montréal

GUOFANG WEI, UC Sabta Barbara

A Sharp Diameter-Dependent Lower Bound for the First Nonzero Neumann Eigenvalue of Geodesic Triangles in Space Forms

We prove a sharp lower bound for the first nonzero Neumann eigenvalue of geodesic triangles of given diameter in two-dimensional space forms. The bound is given by the first positive radial Neumann eigenvalue of a one dimensional model; it is approached by degenerating isosceles triangles. When $K > 0$ and $D = \pi/(2\sqrt{K})$, equality is attained precisely by birectangular triangles. We also prove a hot-spots theorem for non-acute spherical triangles of diameter at most $\pi/2$, and establish antisymmetry and eigenvalue monotonicity for isosceles spherical triangles of diameter $\pi/2$. This is a joint work with Shoo Seto and Yusen Xia.