THOMAS BECK, Fordham University
Nodal domain estimates for dumbbell domains

In this talk, we will discuss the nodal geometry of Neumann eigenfunctions of dumbbell domains, as the width of the thin neck joining the two sub-domains shrinks to zero. We will see that, generically, this forces the number of nodal domains to grow with the eigenvalue, and that in particular the third eigenfunction is Courant sharp (attains equality in the Courant nodal domain theorem) for sufficiently thin necks. In a symmetric case, we will also describe a criterion guaranteeing the existence of pairs of eigenfunctions that are Courant sharp. The proofs of these results require the leading order, non-zero, asymptotics of these eigenfunctions in each part of the dumbbell, which is established via a matched asymptotic analysis with logarithmic terms.

This is based on joint work with Yaiza Canzani, Andrew Lyons, and Jeremy Marzuola.