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Rayleigh's conjecture for vibrating plates: simpler planar proof and nonlinear extensions

Which shape of clamped plate achieves the lowest fundamental tone? Rayleigh conjectured that among planar plates of fixed area, the disk minimizes the first eigenvalue of the bilaplacian. This fourth-order analogue of the Faber–Krahn theorem is substantially harder than the membrane problem, and was not proved in two dimensions until 1995, by Nadirashvili.

I will present a new and simpler proof, based on a paired comparison principle for Poisson equations with sign-changing data. The method extends naturally to a whole family of nonlinear clamped-plate problems: the disk minimizes the nonlinear Rayleigh quotient

$$\frac{\|\Delta u\|_{L^q(\Omega)}}{\|u\|_{L^p(\Omega)}}, \qquad u = \partial_\nu u = 0 \quad \text{on } \partial\Omega,$$

for every $1 \le p, q \le \infty$. Thus the original linear spectral conjecture opens the door to a much larger nonlinear universe.

(Joint work with Dorin Bucur.)