Nonlinear Waves and Dispersive PDEs: Recent Advances in Theory and Computations
Ondes non-linéaires et EDP dispersives: Avancées récentes théoriques et calculatoires
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Overhanging waves in the water wave equations with constant vorticity

In this talk, we present a computer-assisted methodology for proving the existence of periodic traveling gravity waves at the free surface of water, in a flow of constant vorticity over a flat bed. Using conformal mappings, the free-boundary problem is reformulated as a quasilinear pseudodifferential equation for a periodic function of a single variable. We then expand the solution in a Fourier series, reducing the problem to an infinite-dimensional system of equations for the Fourier coefficients.

To establish existence, we employ a Newton–Kantorovich type argument, proving the existence of a true solution in a neighborhood of a numerically computed approximation. The verification of the hypotheses of this fixed-point argument relies on a careful combination of analytical estimates and rigorous numerical computations.

This approach enables us to rigorously prove the existence of an overhanging wave, that is, wave profiles that are no longer graphs of functions. Moreover, it allows for the construction of branches of solutions parameterized by the relative mass flux, bifurcating from the trivial flat solution. In particular, we rigorously construct a branch that originates at the flat solution and extends to a wave with a self-intersecting profile, passing through overhanging configurations along the way.

This is a joint work with Susanna Haziot (Princeton).

JOEL DANHE, University of Minnesota

STEPHEN GUSTAFSON, UBC

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JEAN-PHILIPPE LESSARD, McGill University

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DMITRY PELINOVSKY, McMaster University
Nonlinear stability of periodic waves in the Benjamin-Ono equation

We construct the Lyapunov functional for the periodic waves of the Benjamin–Ono equation based on the conservation of momentum, energy, and a higher-order energy, which exists due to integrability. Positivity of the second variation of the Lyapunov functional for perturbations in Sobolev space $H^1(\mathbb{R})$ is established by utilizing the basis properties of eigenfunctions of the linearized Benjamin–Ono equation in $L^2(\mathbb{R})$. Coercivity of the Lyapunov functional for co-periodic and multiple-periodic perturbations yields the nonlinear orbital stability of the periodic waves in the periodic domain.

ALAN RIQUIER, University of Toronto
A Hamiltonian Description of Overturning Water Waves

Extending the ideas of Craig (2017), we propose a geometric generalization of the Water Waves equations (the Zakharov-Craig-Sulem formulation of Euler's free-surface equations) in physical dimension 2 or 3 where the fluid's free surface is represented by a parametrized curve or surface instead of the graph of some function, thus providing a new model for possibly-overturning, inviscid and irrotational oceanic surface waves. Advection of the free surface is achieved in a Lagrangian manner.

We also introduce a novel variational principle with prescribed tangential velocity for breaking waves, along with the corresponding complete Hamiltonian structure describing both normal and tangential movements of the free surface.

JAMES ROWAN, UT Austin

CATHERINE SULEM, University of Toronto

THIERRYLAURENS, University of Michigan

KATJA VASSILEV, University of Chicago
Derivations of one-dimensional kinetic equations

Wave kinetic equations give an effective description of dispersive systems. In this talk, we look at various 1D dispersive systems and show how a diagrammatic expansion of the solution to the dispersive system gives a framework for these derivations. In particular, we discuss the features of these dispersive systems which restrict the timescale to which these derivations can currently be done. Such features include the dimension of the system, resonances in the nonlinearity, and lack of invariances.

PENG YAN, Concordia University