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Machine learning-assisted moment closure model for the linear Boltzmann equation

The Boltzmann equation, a fundamental equation in kinetic theory, serves as a bridge between microscopic particle dynamics and macroscopic continuum mechanics. However, deriving closed macroscopic moment systems from the Boltzmann equation remains a long-standing challenge due to the intrinsic non-closure of the moment hierarchy. In this talk, we introduce a machine learning (ML)-assisted moment closure model for the linear Boltzmann equation, addressing both the deterministic and stochastic settings. Our approach leverages neural networks to learn the spatial gradient of the unclosed highest-order moment, enabling effective training through natural output normalization. For the deterministic problem, to ensure global hyperbolicity and stability, we derive and apply the constraints that enforce symmetrizable hyperbolicity of the system. For the stochastic problem, we adopt the generalized polynomial chaos (gPC)-based stochastic Galerkin method to discretize the random variables, resulting in a system for which the approach in the deterministic case can be used similarly. Several numerical experiments are shown to demonstrate the effectiveness and accuracy of our ML-assisted moment closure model for the linear Boltzmann equation with or without uncertainties.