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Delay-driven oscillatory dynamics in an avian influenza model with dose-dependent environmental transmission

Avian influenza remains a major transboundary infectious disease affecting poultry and wild birds, with important consequences for food security, agricultural economies, and public health. Environmental persistence of avian influenza viruses and dose-dependent infection risk introduce transmission mechanisms that are not fully captured by classical direct-transmission models. We develop and analyse a nonlinear delay differential equation model incorporating generalized dose-dependent environmental transmission. Positivity and boundedness are established, and the disease-free and endemic equilibria are characterized. The basic reproduction number is derived, and local stability is investigated through the associated characteristic equations. Analytical stability-switch conditions and numerical continuation show that increasing the environmental delay can destabilize the endemic equilibrium through a Hopf bifurcation. DDE-BIFTOOL computations and an independent Beretta–Kuang stability-switch analysis identify the same critical delay and oscillation frequency, providing consistent verification of the bifurcation threshold. Computation of the first Lyapunov coefficient further shows that the Hopf bifurcation is subcritical, with an unstable branch of periodic solutions emerging on the stable-equilibrium side of the critical point. Numerical simulations reveal increasingly persistent oscillatory transients as the delay approaches the Hopf threshold. Near the unstable periodic branch, perturbations in environmental viral decay, shedding, and dose-response steepness substantially influence whether oscillations grow or decay. These results demonstrate how environmental transmission delays and dose-dependent exposure can generate complex oscillatory dynamics in avian influenza transmission and highlight parameter regimes in which small changes in environmental processes may strongly affect epidemic behaviour.