Higher Category Theory Across Mathematics
La théorie des catégories supérieurs à travers les mathématiques
(Org: **Kristine Bauer** (University of Calgary) and/et **Martina Rovelli** (University of Ottawa))

JULIE BERGNER, University of Virginia
Classifying diagrams and equivariant operads

One description of the composition product for operads can be formulated elegantly in terms of the component spaces of the classifying diagram associated to a small category. The classifying diagram was originally developed as one way to regard a category as a complete Segal space, and can be thought of as a refinement of the ordinary nerve construction. In ongoing work with Calle, Chan, Osorno, and Sarazola, we want to develop a similar approach to a composition product for equivariant operads. For this purpose, we can use a similar construction using certain kinds of quotient maps in place of isomorphisms. We hope this construction will have a broader use in other equivariant constructions.

ELEFTHERIOS CHATZITHEODORIDIS, Virginia Commonwealth University
Models for rational $(\infty, 1)$-categories

An $(\infty, 1)$-category is a category enriched in spaces, possibly weakly. Our understanding of $(\infty, 1)$-categories has been advanced thanks to the development of various models for $(\infty, 1)$-categories, that is, mathematical objects that exhibit the structure of an $(\infty, 1)$-category. Two such models are complete Segal spaces, as introduced by Rezk, and Segal categories, as developed from the homotopical perspective by Bergner.

We introduce rational $(\infty, 1)$-categories, which are $(\infty, 1)$-categories enriched in spaces whose higher homotopy groups are rational vector spaces. We produce two models for rational $(\infty, 1)$-categories, rational complete Segal spaces and rational Segal categories. Our argument works for enrichment in general localizations of spaces, such as the v_n-periodic spaces of Heuts in unstable chromatic homotopy theory.

ROBIN COCKETT, University of Calgary

GEOFF CRUTTWELL, Mount Allison University

PEDRO BRUNIALTI LIMA DE ANDRADE, University of Virginia
Involutions on (∞, n)-categories of Bordisms and Reflectopm TQFTs

Formally, a topological quantum field theory (TQFT) is a symmetric monoidal functor from a category of bordisms into an appropriate target category. Depending on the choice of bordism category, the theory may carry extra structure compatible with the monoidal structure — namely, an involution. By involution, we mean a $\mathbb{Z}/2$ action on the bordism category which is compatible with the monoidal structure. In this talk, I will outline how to get these actions on the (∞, n)-categories of bordisms defined by Calaque and Scheimbauer, and I will define reflection TQFTs, in the sense of Freed and Hopkins, in this setting. Furthermore, I will exemplify what kind of obstructions a reflection structure imposes on the point values of a theory.

ARGHAN DUTTA, University of Ottawa
Symmetric monoidal (∞, n)-categories

Symmetric monoidal (∞, n)-categories have been considered in a variety of forms and from several perspectives, including Lurie's commutative algebra objects and, for $(\infty, 1)$-categories, coCartesian fibrations over finite pointed sets satisfying a Segal condition; Gepner and Haugseng's iterated enrichment; and the special Γ-objects of Schommer-Pries, Calaque–Scheimbauer, and Johnson-Freyd.

In this talk, I will introduce a definition in terms of left double (∞, n)-fibrations, building on the framework for double higher categories developed by Moser, Rasekh, and Rovelli. Working internally in a suitable model category presenting (∞, n)-categories, we express the symmetric monoidal Segal condition through spans associated with the inert maps of finite pointed sets. I will then focus on the quasi-categorical case, where we prove a Quillen equivalence with Lurie's presentation via coCartesian fibrations; in the strict one-dimensional case, our approach recovers the ordinary symmetric monoidal categories. Our comparison uses strings of coCartesian morphisms and a staircase construction to pass between the two descriptions. I will explain the ideas behind these constructions and how they relate the two Segal conditions for symmetric monoidality.

MARTIN FRANKLAND, University of Regina
Near-ringoids of operations

When we consider unary operations acting on homotopy groups or cohomology groups, there's an issue with the algebra of operations: it's not an algebra. (a) Operations between different degrees give a "many objects" structure; that's the "oid" part. (b) The presence of non-additive operations means that composition is only linear in one variable rather than bilinear; that's the "near" part. We will discuss some of the algebraic structure of near-ringoids, as well as examples from homotopy theory. Based on joint work with Manak Singh.

ADINA GOLDBERG, Fields Institute
A double category of quantum sets, functions and relations

We introduce the double category $\mathbb{Q}$**Rel** of finite quantum sets, functions, and binary relations, building on the work of Weaver, Kornell, and Musto-Reutter-Verdon. It is a candidate for a †-compact double category in the sense that there is a † structure on squares that is compatible with the duals structure. In analogy to the classical case of the double category $\mathbb{R}$**el** of functions and relations, the 2-categories **QSet** of finite quantum sets and quantum functions, and **qRel** of finite quantum sets and binary relations arise as the horizontal and vertical 2-categories of $\mathbb{Q}$**Rel**.

Time permitting, we illustrate how $\mathbb{Q}$**Rel** models both new and existing quantizations of some constructions from classical graph theory and quantum information theory.

STEFANO LUNEIA, University of Ottawa

DORETTE PRONK, Dalhousie University

CARLO MARIA SCANDOLO, University of Calgary
A general type-theoretic approach to higher-order quantum resources

Quantum resources exist in a hierarchy of multiple levels. At order zero, quantum states are transformed by linear maps (channels, or gates) in order to perform computations or simulate other states. At order one, gates and channels are transformed by linear maps (superchannels) in order to simulate other gates. To develop a full hierarchy of quantum resources, beyond those first two orders, and to account for the fact that quantum protocols can interconvert resources of different orders, we need a theoretical framework that addresses all orders in a uniform manner. We introduce a framework based on a system of types, which label the different kinds of objects that are present at different orders. Subsequently, we equip the framework

with a parallel product operation that modifies and generalizes the tensor product so as to be operationally meaningful for maps of distinct and arbitrary orders.

PRIYAA SRINAVASAN, Tallinn University of Technology

KIMBALL STRONG, Cornell University

DANIEL TEXEIRA, Dalhousie University

DAVID WHITE, Queen's University

INNA ZAKHAREVICH, Cornell University