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On the Gaussian Torsion

The Ornstein–Uhlenbeck operator is the natural analogue of the Laplacian on $\mathbb{R}^n$ equipped with the standard Gaussian measure. Although it is not translation invariant, it shares many features with the Laplace operator. In this talk, we consider the torsion function and torsional rigidity associated with the Ornstein–Uhlenbeck operator. In the classical Euclidean setting, torsional rigidity satisfies a Brunn–Minkowski inequality on convex domains. Moreover, the square root of the torsion function is concave on every convex domain, while the torsion function itself is concave under suitable smoothness and curvature assumptions on the boundary. In joint work with Xuan Hien Nguyen, we show that the Gaussian setting is strikingly different. The natural Brunn–Minkowski-type inequality conjectured for Gaussian torsional rigidity fails in every dimension greater or equal two, even for centrally symmetric, smooth, strictly convex domains. We also construct, in every such dimension, centrally symmetric, uniformly convex smooth domains whose Ornstein–Uhlenbeck torsion function is not concave.