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Beyond the unique pair: computations with rational triangles of equal perimeter and area

Can two non-congruent rational triangles have the same perimeter and the same area? A surprising result of Hirakawa and Matsumura shows that, if one triangle is right and the other is isosceles, there is a unique such pair up to homothety.

We ask what happens when the right angle is replaced by a prescribed angle θ with rational cosine ρ . This leads to a family of genus-2 curves, so that questions about existence and uniqueness of triangle pairs become questions about rational points. I will describe how explicit arithmetic geometry and computation, including 2-descent and Chabauty, give uniqueness results for many values of ρ . On the other hand, a two-parameter construction leads to a dense set of acute angles with rational cosine admitting at least two distinct pairs, examples with at least six distinct pairs, and an elliptic $K3$ surface arising from the question of when a fixed scalene θ -triangle can have two isosceles partners.

This is joint work with Jesse Allen, Olivier Beaudry-Ogden, Théophane Bouchard, Matéo Frély, Abel-Jimmy Oyono-Montoki, and Berend Ringeling.