KEVIN ANDERSON, University of Ottawa

Towards a Second Fundamental Theorem for $U_q(\mathfrak{gl}_n)$

Classical invariant theory studies the elements of a polynomial algebra which are invariant under a given group action. An answer to this problem consists of two parts: a "First Fundamental Theorem" (FFT) of invariant theory describes the generators of the invariant subalgebra, and a "Second Fundamental Theorem" (SFT) describes the relations among these generators. The same problem can be posed in other contexts in algebra and representation theory. For instance, one can replace the group with a Hopf algebra and consider actions (and coactions) on a quantized polynomial algebra. Recent work by Letzter–Sahi–Salmasian (2024) has established an "operator commutant" FFT for the quantized enveloping algebra $U_q(\mathfrak{gl}_n)$ in such a setting. The relevant polynomial algebra considered is a quantized Weyl algebra, which is constructed as a deformed twisted tensor product and comes with a natural $U_q(\mathfrak{gl}_m) \otimes U_q(\mathfrak{gl}_n)$-module algebra structure. In this case, the invariant subalgebra is generated by certain homogeneous degree 2 polynomials called quantized polarization operators. Our present research aims to better understand the structure of this invariant subalgebra, first and foremost by determining the commutation relations among these polarization operators.