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On Equivalent and Strengthened Affine Fractional Sobolev Inequalities

Affine fractional Sobolev inequalities provide refinements of classical fractional Sobolev inequalities by incorporating affine geometric structures. We establish several equivalent formulations of these inequalities. In particular, we show that affine fractional Sobolev inequalities, up to multiplicative constants, are equivalent to affine fractional Lorentz, Hardy, and isocapacitary inequalities.

We introduce affine fractional capacities and establish their fundamental properties, including capacitary strong-type inequalities. Using these capacities, we further strengthen the affine fractional Sobolev inequalities through embeddings into capacitary Lorentz spaces. As applications, we derive affine fractional Gagliardo–Nirenberg inequalities and affine fractional logarithmic Sobolev inequalities.

Our results highlight the interplay between harmonic analysis, affine geometry, fractional nonlocal regularity, and capacity theory, providing new perspectives on the refinement and equivalence of functional inequalities.