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Non-concentration estimates for Laplace eigenfunctions on compact C^∞ manifolds with boundary

Let Ω be an n-dimensional compact Riemannian manifold with C^∞ boundary, and consider L^2-normalized eigenfunctions $-\Delta\phi_\lambda = \lambda^2\phi_\lambda$ with Dirichlet or Neumann boundary conditions. In this talk, I will describe reccent joint work with H. Christianson (UNC) on the extension of interior nonconcentration bounds up to the boundary. Specifically, using purely stationary local methods, we prove that for such Ω it follows that for *any* $x_0 \in \overline{\Omega}$ (including boundary points) and for all $\mu \geq C_\Omega h$ with sufficiently large constant $C_\Omega > 0$,

$$\|\phi_\lambda\|^2_{B(x_0,\mu)\cap\Omega} = O(\mu). \tag{1}$$

We also extend a result of C. Sogge to manifolds with smooth boundary and show that when $n \geq 3$,

$$\|\phi_\lambda\|_{L^\infty(\Omega)} \leq C\lambda^{\frac{n}{2}} \cdot \Big(\sup_{x\in\Omega} \|\phi_\lambda\|_{L^2(B(x,\lambda^{-1})\cap\Omega)} \Big). \tag{2}$$

The sharp sup bounds $\|\phi_\lambda\|_{L^\infty(\Omega)} = O(\lambda^{\frac{n-1}{2}})$ for Dirichlet or Neumann eigenfunctions proved by D. Grieser are then an immediate consequence of these bounds.