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Dimensions of type A Hessenberg varieties over a fixed sheet

This talk is on joint work with Martha Precup and Colleen Robichaux.

Hessenberg varieties $\mathcal{H}_{\text{ess}}(X, \mathbf{h})$ are subvarieties of the flag variety parameterized by a Hessenberg function $\mathbf{h} : [n] \rightarrow [n]$ and a matrix $X \in \mathfrak{gl}_n(\mathbb{C})$. Hessenberg varieties are known to be connected to the theory of quasisymmetric functions through the study of their cohomology rings. In recent work, Goldin and Precup showed the existence of flat degenerations of Hessenberg varieties to nilpotent Hessenberg varieties over the minimal sheet. This implies that all Hessenberg varieties over the minimal sheet have the same dimension. We generalize this dimension result to arbitrary sheets. Specifically, we prove that for a fixed Hessenberg function $\mathbf{h} : [n] \rightarrow [n]$, all Hessenberg varieties $\mathcal{H}_{\text{ess}}(X, \mathbf{h})$ defined in the type A flag variety by linear operators X from the same sheet of the Lie algebra $\mathfrak{gl}_n(\mathbb{C})$ have the same dimension.