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Honeycombs and Sums of Hermitian Matrices, Revisited

The Horn Problem is a classic problem, dating back to at least 1912. If A and B are two Hermitian matrices with known eigenvalue spectra, it asks about the possible spectra of $A+B$. The problem was fully resolved by Knutson, Tao, and Klyachko using tools from Schubert calculus and representation theory. An important component of the solution was honeycombs, a combinatorial model for Littlewood-Richardson coefficients developed by Knutson and Tao. Their key theorem states that a honeycomb with given boundary values exists if and only if there exists a triple $(A, B, A + B)$ of Hermitian matrices with corresponding eigenvalue spectra. This theorem gives a pleasant combinatorial and geometric answer to the original Horn problem. We present a new proof of this bridge between honeycombs and Hermitian matrices. The main surprise is that the proof can be made very elementary. It bypasses Schubert calculus and representation theory, making use only of linear algebra, basic calculus, and combinatorics. Our proof also suggests new perspectives on Littlewood-Richardson coefficients. This project is joint work with Ankur Moitra and Alex Postnikov.