Analytic and Combinatorial Aspects in Convex and Discrete Geometry
Aspects analytiques et combinatoires en géométrie convexe et discrète
(Org: **Károly Bezdek** (University of Calgary, Canada), **Fodor Ferenc** (University of Szeged, Hungary and University of Calgary, Canada) and/et **Alina Stacu** (Concordia University, Canada))

MARIA ALFONSECA, North Dakota State University
On planar bodies of variable density that float in equilibrium in all orientations

In 1935, Ulam asked if a convex body of uniform density which floats in equilibrium in every orientation must be a sphere. The answer has been shown to be negative for bodies of constant density 1/2. In this talk we show that any star body may be made to float in equilibrium in all orientations if it has a specific variable density, which we determine explicitly. We also consider the case, in two dimensions, of a non-compactly supported, non-radial density that achieves the same result. This is joint work with Stephanie Juntunen.

GERGELY AMBRUS, University of Szeged, Hungary

KAROLY BEZDEK, University of Calgary

KÁROLY J. BÖRÖCZKY, Alfréd Rényi Mathematical Research Institute, Hungary

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DMITRY FAIFMAN, University of Montreal

FERENC FODOR, University of Szeged, Hungary

KYLE FRIDBERG, Cornell University

BALÁZS GRÜNFELDER, University of Szeged, Hungary
Intrinsic volume moments of random beta-prime polytopes

We consider random beta-prime polytopes, which are convex hulls of i.i.d. beta-prime distributed random points in $\mathbb{R}^d$. After a suitable scaling, these polytopes converge in distribution to the convex hulls of Poisson point processes with power-law intensity density functions. We prove convergence of moments for the intrinsic volumes of random beta-prime polytopes.

We also consider a related spherical model in the upper open hemisphere of S^d. We choose i.i.d. random points in S^d_+ according to the uniform distribution or certain other rotationally symmetric distributions around the north pole of the hemisphere. The

spherical convex hull of the random points is a spherical random polytope. After gnomonic projection, the push-forwards of such polytopes are beta-prime polytopes. We prove moment convergence of the spherical volume of the spherical random polytopes.

Joint work with Ferenc Fodor and Péter Kevei (both from the University of Szeged).

JULIÁN HADDAD, Universidad de Sevilla

Approximate counterexamples to the Barker–Larman problem in dimension 4

Consider a convex body $K \subseteq \mathbb{R}^n$ containing the unit Euclidean ball B, and the hyperplanes H tangent to B.

The Barker–Larman problem asks whether K must be a Euclidean ball if the sections by all these hyperplanes H have constant $n-1$ dimensional volume.

The problem for bodies of revolution in dimension $n = 4$ has a very special structure, and the answer is very likely NO.

We show a way to construct approximate counterexamples as solutions to a bifurcation equation in the space of periodic functions.

PAVLOS KALANTZOPOULOS, University of Waterloo

Commutativity in a Poincaré-Type Inequality

Can a geometric test reveal whether two matrices commute? Motivated by the log-Brunn–Minkowski conjecture, we study the Poincaré-type inequality

$$\mathrm{Var}_{\mu_{K,\Sigma}}\left(\langle A\Sigma^{-1}y, y\rangle\right) \leq \int_{\mathbb{R}^n} \left\langle \left(A^2\Sigma^{-1} + A\Sigma^{-1}A\right) y, y\right\rangle \, d\mu_{K,\Sigma}(y).$$

Here A is symmetric, and $\mu_{K,\Sigma}$ is the centered Gaussian measure with positive-definite covariance Σ, restricted and normalized on an origin-symmetric convex body K. We show that the inequality holds for every origin-symmetric orthogonal box K exactly when A and Σ commute. However, the inequality holds without commutativity if K is unconditional with respect to an orthonormal eigenbasis of Σ.

BERNARDO GONZÁLEZ MERINO, University of Murcia, Spain

STEPHANIE MUI, GeorgiaTech

SERHII MYROSHNYCHENKO, University of the Fraser Valley

Two Centres of Mass, One Sharp Bound

In their book "Unsolved Problems in Geometry", Croft, Falconer, and Guy asked how far apart the centroids of a planar convex body and its boundary can be relative to the body's width or diameter (Problem A25). They also raised the question in higher dimensions. Nazarov, Ryabogin, and Yaskin settled the planar case and conjectured the optimal bound in all dimensions.

We prove this conjecture. For every convex body $K \subset \mathbb{R}^n$, $n \geq 2$, and every unit vector u, the volume centroid $c(K)$ and the surface-area centroid $c(\partial K)$ satisfy

$$\left|\langle c(\partial K) - c(K), u\rangle\right| \leq D_n \, w_K(u),$$

where $w_K(u)$ is the width of K in direction u. The constant D_n is optimal in both this inequality and the resulting diameter bound. The same sharp constant appears in the analogous width bound for the distance between the orthogonal projection of $c(K)$ onto a hyperplane and the centroid of the projected body.

The proof compares uniform and length-weighted averages of the midpoints of parallel chords. The Kannan–Lovász–Simonovits localization lemma reduces the comparison to a one-dimensional inequality. A family of polytopes converging to a line segment establishes sharpness.

This is joint work with Dmitry Ryabogin, Kateryna Tatarko, and Vladyslav Yaskin.

BRANDON OLIVA, Case Western Reserve University
Illumination bodies for ball-convex bodies.

We introduce illumination bodies in the class of n-dimensional R-ball convex bodies. We will prove that the illumination bodies are convex, and that a left derivative of volume gives rise to a surface area measure for ball-convex bodies, the relative surface area measure. This talk is based on a paper titled "Illumination bodies for ball-convex bodies" by Elisabeth Werner, Diliya Yalikun, and Brandon Oliva.

LEI QIN, CRM Montreal

ADAM SAGMEISTER, University of Szeged, Hungary

ETHAN SAUNDERS, University of Calgary

EGON SCHULTE, Northeastern University

VIKTOR VÍGH, University of Szeged, Hungary

TINGZHOU YU, University of Alberta

SHIRA ZERBIB, Iowa State University
Piercing convex sets with hyperplanes

We discuss the number of hyperplanes needed to pierce a finite family of convex sets in $\mathbb{R}^d$ with a given matching number or other local intersection properties.