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Two Centres of Mass, One Sharp Bound

In their book "Unsolved Problems in Geometry", Croft, Falconer, and Guy asked how far apart the centroids of a planar convex body and its boundary can be relative to the body's width or diameter (Problem A25). They also raised the question in higher dimensions. Nazarov, Ryabogin, and Yaskin settled the planar case and conjectured the optimal bound in all dimensions.

We prove this conjecture. For every convex body $K \subset \mathbb{R}^n$, $n \geq 2$, and every unit vector u, the volume centroid $c(K)$ and the surface-area centroid $c(\partial K)$ satisfy

$$\big|\langle c(\partial K) - c(K), u\rangle\big| \leq D_n \, w_K(u),$$

where $w_K(u)$ is the width of K in direction u. The constant D_n is optimal in both this inequality and the resulting diameter bound. The same sharp constant appears in the analogous width bound for the distance between the orthogonal projection of $c(K)$ onto a hyperplane and the centroid of the projected body.

The proof compares uniform and length-weighted averages of the midpoints of parallel chords. The Kannan–Lovász–Simonovits localization lemma reduces the comparison to a one-dimensional inequality. A family of polytopes converging to a line segment establishes sharpness.

This is joint work with Dmitry Ryabogin, Kateryna Tatarko, and Vladyslav Yaskin.