PAVLOS KALANTZOPOULOS, University of Waterloo
Commutativity in a Poincaré-Type Inequality

Can a geometric test reveal whether two matrices commute? Motivated by the log-Brunn–Minkowski conjecture, we study the Poincaré-type inequality

$$\mathrm{Var}_{\mu_{K,\Sigma}}\left(\left\langle A\Sigma^{-1}y, y\right\rangle\right) \leq \int_{\mathbb{R}^n} \left\langle \left(A^2\Sigma^{-1} + A\Sigma^{-1}A\right) y, y\right\rangle \, d\mu_{K,\Sigma}(y).$$

Here A is symmetric, and $\mu_{K,\Sigma}$ is the centered Gaussian measure with positive-definite covariance Σ, restricted and normalized on an origin-symmetric convex body K. We show that the inequality holds for every origin-symmetric orthogonal box K exactly when A and Σ commute. However, the inequality holds without commutativity if K is unconditional with respect to an orthonormal eigenbasis of Σ.