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Image Cones and Conjugacy Invariants of Matrix Lie Subalgebras

The classification of Lie subalgebras up to conjugacy is a fundamental problem in Lie theory. An important aspect of this problem is finding invariants that distinguish non-conjugate subalgebras.

In this talk, I present results concerning the *image cone* of a matrix Lie subalgebra $\mathfrak{g}$, defined by

$$\mathrm{im}(\mathfrak{g}) = \{Xv : X \in \mathfrak{g},\ v \in \mathbb{C}^n\}.$$

Although the image cone is closed under scalar multiplication, it need not be a vector subspace. I investigate conditions under which it is a subspace, proving that, among all conjugacy classes of subalgebras of $\mathfrak{sl}_3(\mathbb{C})$, exactly one has an image cone that is not a vector subspace. I also establish that the image cone is a vector subspace for every subalgebra of $\mathfrak{so}_n(\mathbb{C})$ when $n \leq 4$, while this property does not necessarily hold in higher dimensions.

In addition, I introduce and study conjugacy invariants, including the total column rank and total row rank. Together with other structural invariants, these provide a complete set of invariants distinguishing the conjugacy classes of subalgebras of $\mathfrak{sl}_3(\mathbb{C})$.