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Lamé monodromy and modular Schwarzian equations

We introduce two second-order ordinary differential equations in Schrödinger form: the Lamé equation in the elliptic variable (z), with potential $(n(n+1)\wp(z;\tau)+B)$, and a modular differential equation in the modulus (τ), with a weight-four modular potential. We link the two equations through the period monodromy of the Lamé equation. At spectral values where the period monodromy is not completely reducible, we identify its projective Jordan datum with a projective period class $\mathcal{C}$ and show that $\mathcal{C}$ is equivariant under the modular group.

We then study the permutation monodromy obtained by varying the elliptic modulus. Over the Legendre parameter line, each nonlinear irreducible Type I or marked Type II spectral cover of degree N is simply branched at $N(N-1)$ points and has full symmetric monodromy S_N. We explain how intersections of the marked branch loci govern the Galois groups, including the exceptional quadratic cases.

We will close with the explicit example $n=3$, $B=0$, where the Schwarzian derivative of the projective period class satisfies

$$\{\mathcal{C},\tau\} = 2\pi^2 E_4(\tau).$$