Analytic–Geometric Synergies: Harmonic Analysis and Convexity
Synergies analytiques et géométriques : analyse harmonique et convexité

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ALMUT BURCHARD, University of Toronto

Strict concavity properties of cross covariograms

The cross covariogram of two convex bodies K, L is the volume of the intersection of K with v+L, as a function of the translation vector v. This function is known to be log concave; here I will present precise conditions for *strict* log concavity. (Joint work with Gabriele Bianchi and Lawrence Lin)

BLAIR DAVEY, Montana State University

Self-similar sets and Lipschitz curves

If E is a purely unrectifiable 1-set in the plane, then the intersection of E with any Lipschitz graph has zero 1-dimensional Hausdorff measure. This leads to a natural question: Given a purely unrectifiable 1-set, can we find a Lipschitz curve for which the intersection with E is non-trivial in some dimension less than 1? Going further, how close to 1 can we get? We discuss the answer to this question for self-similar sets. This talk covers joint work with Silvia Ghinassi and Bobby Wilson.

DMITRY FAIFMAN, Université de Montréal

Tubes and valuations in Lie groups

Evaluating the volume of metric tubes around submanifolds traces back to Weyl's famous resolution of Hotelling's problem in Euclidean space. Further results were obtained by A. Gray and others, particularly in rank one symmetric spaces. We consider a Lie group equipped with a bi-invariant Riemmanian (or more generally, Finslerian) metric. Utilizing Alesker's theory of valuations on smooth manifolds, in particular the convolution of valuations on compact Lie groups introduced by Alesker and Bernig, and borrowing tools and ideas from deformation theory, we give an explicit power series for the volume of a tube around a submanifold. Based on a joint work with A. Bernig and J. Kotrbaty.

FERENC FODOR, University of Szeged, Hungary

Central diagonal sections of Gaussian cubes

We consider Gaussian-type probability measures in the standard n-dimensional cube and study the induced measure of hyperplane sections through the origin and orthogonal to a main diagonal. Using a formula proved by König and Koldobsky (2013), we investigate the asymptotic behaviour of the measure of sections as n tends to infinity. Joint work with Bernardo Gonzalez Merino (University of Murcia, Spain).

RYAN GIBARA, Cape Breton University

The Neumann problem in metric measure spaces

In this talk, we will discuss progress on the study of the Neumann problem for the p-Laplacian in the context of a doubling metric measure space supporting a p-Poincaré inequality. The focus will be on recent joint work with Luca Capogna, Rikka Korte, and Nageswari Shanmugalingam, where we establish Hölder regularity of the weak solutions with an exponent that is sharp with respect to the hypotheses we require from the Neumann data.

PAUL HAGELSTEIN, Baylor University
Current developments in the theory of differentiation of integrals

We will provide an overview of current developments in the theory of differentiation of integrals. Particular emphasis will be placed on a recent result, extending prior work of Bateman and Katz, that provides a condition on directional maximal operators on $\mathbb{R}^2$ sufficient to ensure that they are unbounded on $L^p(\mathbb{R}^2)$ for $1 \leq p < \infty$. This recent work is joint with Blanca Radillo-Murguia and Alex Stokolos.

ALEX IOSEVICH, University of Rochester
The Fourier ratio, probabilistic method and signal recovery

We are going to discuss the ratio of the L1 to L2 norms of the Fourier transform in a variety of different contexts as a proxy for the complexity of a signal. We are going to see that if the Fourier ratio is small, the signal can be well-approximated by a trigonometric polynomial of a low degree. Applications to signal recovery and connections with restriction theory of the Fourier transform will be discussed.

DMITRY JACOBSON, McGill
Extremal metrics on graphs

We review several old and new results about extremal metrics for various graph functionals.

PAVLOS KALANTZOPOULOS, University of Waterloo
A multiversion of real and complex hypercontractivity.

We establish a multiversion of real and complex Gaussian hypercontractivity. Our result generalizes Nelson's hypercontractivity in the real setting and the works of Beckner, Weissler, Janson, and Epperson in the complex setting to several functions. The proof relies on heat semigroup methods, where we construct an interpolation map that connects the inequality at the endpoints. As a consequence, we derive sharp multiversion of the Hausdorff-Young inequality and the log-Sobolev inequality. This is joint work with Paata Ivanisvili.

DYLAN LANGHARST, Carnegie Mellon University
Grünbaum's inequality for probability measures

Given a convex body K in $\mathbb{R}^n$, a natural question is: if one partitions the body into two pieces along its barycenter, how small can each piece be? By "partition along its barycenter", we mean intersecting K with a half-space whose boundary is a hyperplane containing said barycenter. Grünbaum showed that the volume of each piece is at least $\left(\frac{n}{n+1}\right)^n$ times the total volume of K. Furthermore, this constant is sharp: there is equality if and only if K is a cone, which means there exists a $(n-1)$-dimensional convex body L and a vector b, such that K has face L and vertex b (i.e. K is the convex hull of b and L).

In this work, which is joint with M. Fradelizi, J. Liu, F. Marin Sola, and S. Tang, we are interested in generalizing Grünbaum's inequality to other measures. Our main results are a sharp inequality for the Gaussian measure and a sharp inequality for s-concave probability measures. The characterization of the equality case is of particular interest. Along the way, we discover new facts about the equality case of the Borell-Brascamp-Lieb inequality.

LIANGBING LUO, Queen's University
Logarithmic Sobolev inequalities on some infinite-dimensional groups

The logarithmic Sobolev inequality has been first introduced and studied by L. Gross on a Euclidean space, and since then it found many applications. In particular, many existing results concern the question on how the constant in the logarithmic

Sobolev inequality depends on the geometry of the underlying space. As for many of such infinite-dimensional groups curvature bounds (or classical Bakry-Emery estimates) are not available, we use different techniques. Examples are provided.

MARCU-ANTONE ORSONI, Université Laval
On the dimension of observable sets for the heat equation

Let Ω be a bounded C^1 domain in $\mathbb{R}^n$. If $\omega \subset \Omega$ is an open set, it is today well-known that the heat equation is null-controllable (or equivalently observable) from ω. In this talk, I will show that this result still holds when ω is any measurable set with Hausdorff dimension strictly greater than $n-1$. Even if this observability result is sharp with respect to the scale of Hausdorff dimension, we will see how to construct observable sets with codimension greater than 1 and how they are related to nodal sets of Laplace eigenfunctions. Joint work with A.W. Green, K. Le Balc'h and J. Martin.

ANDRIY PRYMAK, University of Manitoba
On asymptotic Lebesgue's universal covering problem

A classical theorem of Jung states that any set of diameter 1 in an n-dimensional Euclidean space is contained in a ball J_n of radius $\sqrt{\frac{n}{2n+2}}$; in other words, J_n is a universal cover in $\mathbb{E}^n$.

Lebesgue's universal covering problem, posed in 1914, asks for the convex set of smallest area that serves as a universal cover in the plane ($n=2$). We show that in high dimensions, Jung's ball J_n is asymptotically optimal with respect to volume: for any universal cover $U \subset \mathbb{E}^n$,

$$\mathrm{Vol}(U) \geq (1-o(1))^n \mathrm{Vol}(J_n).$$

Joint work with A. Arman, A. Bondarenko and D. Radchenko.

YANA TEPLITSKAYA, Paris-Saclay University
About maximal distance minimizers. Regularity and explicit examples

Consider a compact set $M \subset \mathbb{R}^d$ and $l > 0$. A maximal distance minimizer problem is to find a connected compact set Σ of the length (that is, one-dimensional Hausdorff measure $\mathcal{H}^1$) at most l that minimizes

$$\max_{y \in M} \mathsf{dist}(y, \Sigma),$$

where $dist$ stands for the Euclidean distance. In this talk, I will survey known results on maximal distance minimizers, including explicit examples (such as a circle, a rectangle, and a minimizer with an infinite number of corner points), as well as the regularity of their local structure (a finite number of branching points in the plane and at most three tangent rays at any point of a minimizer in any dimension). I will also discuss several open problems in this area.

DENIS VINOKUROV, Université de Montréal
Topological Tensor Products, Harmonic Maps, and Spectral Optimization

In the classical scalar setting, the Rellich–Kondrachov theorem provides compact Sobolev embeddings that are central to many arguments in analysis. This compactness fails for Sobolev maps with values in infinite-dimensional targets, such as Hilbert spaces, and standard direct methods break down.

I will explain how techniques from the theory of topological tensor products allow one to recover a useful compactness framework for certain classes of variational integrals for vector-valued functions. As a key example, we consider Dirichlet-type energies whose critical points are harmonic maps into the infinite-dimensional Hilbert sphere. Their energy densities also arise as critical points of a Laplace eigenvalue optimization problem. We address both the existence of such optimal densities and the regularity of the associated harmonic maps.

ELISABETH WERNER,
Floating bodies for ball-convex bodies

Abstract: We define floating bodies in the class of n-dimensional ball-convex bodies. A right derivative of volume of these floating bodies leads to a surface area measure for ball-convex bodies which we call relative surface area. We show that this quantity is a rigid motion invariant, upper semi continuous valuation.

Joint work with C. Schuett and D. Yalikun.