Additive Combinatorics and Applications
Combinatoire additive et applications

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ERNIE CROOT, Georgia Institution of Technology
A survey of some results on digits of numbers in different bases related to a problem of R. L. Graham

In this talk I'll survey some results related to R. L. Graham's problem about whether there exist infinitely many integers n such that gcd$(\binom{2n}{n}, 105) = 1$, including some results due to myself and collaborators.

ZHENCHAO GE, University of Waterloo
An Additive property for polynomial sequence in function fields

Ruzsa established optimal lower bounds for the natural density $\underline{d}(A + P_k)$ in terms of $\underline{d}(A)$, where $A \subseteq \mathbb{N}$ is a set of small density and P_k denotes the set of kth powers of primes. In this talk, we will introduce a function field analogue of this result. Let $\mathbb{F}$ be a finite field. We show that any weighted polynomial sequence in $\mathbb{F}[t]$ admitting three broad hypotheses satisfies an analogue of Ruzsa's inequality. As a corollary, the inequality holds for degree-k polynomial sequences $(\mathrm{char}(\mathbb{F}) \nmid k)$ over irreducible elements. Notably, our the result extends to the case when the polynomial degree exceeds the characteristic.

MARCEL GOH, McGill University
Block complexity and idempotent Schur multipliers

We call a matrix blocky if, up to row and column permutations, it can be obtained from an identity matrix by repeatedly applying one of the following operations: duplicating a row, duplicating a column, or adding a zero row or column. Blocky matrices are precisely the boolean matrices that are contractive when considered as Schur multipliers. It is conjectured that any boolean matrix with Schur multiplier norm at most γ is expressible as a signed sum

$$A = \sum_{i=1}^{L} \pm B_i$$

for some blocky matrices B_i, where L depends only on γ. This conjecture is an analogue of Green and Sanders's quantitative version of Cohen's idempotent theorem. In this paper, we prove bounds on L that are polylogarithmic in the dimension of A. Concretely, if A is an $n \times n$ matrix, we show that one may take $L = 2^{O(\gamma^7)} \log(n)^2$.

LEO GOLDMAKHER, Williams College and Carnegie Mellon University
Large subsets are sumsets

Large subsets of $[n]$ can be expressed in the form $A+B$ with A, B sets of cardinality at least 2. How large must a subset be to guarantee such an additive decomposition? We show that any subset larger than $(n - \log n)$ must admit such a decomposition. This is nearly optimal: for each n we construct an indecomposable subset of $[n]$ of size larger than $(n - 4\log n)$. We conclude with a discussion of higher-dimensional analogues of this question. This is joint work with a number of former participants in the SMALL summer REU program at Williams College.

DAVID GRYNKIEWICZ, University of Memphis
Towards a Kneser-Pollard Theorem

Let $A, B \subset G$ be finite, nonempty subsets of an abelian group G. For $t \geq 1$, the t-popular sumset $A +_t B$ denotes all $x \in G$ having t distinct tuples $(a_1, b_1), \ldots, (a_t, b_t) \in A \times B$ with $x = a_1 + b_1 = \ldots = a_t + b_t$. When $t = 1$, Kneser's Theorem says

$$|A +_1 B| \geq |A| + |B| - |H|,$$

where $H = \{x \in G : \ x + A + B = A + B\} \leq G$ is the stabilizer of $A + B$. When $|G| = p$ is prime, Pollard's Theorem says

$$\sum_{i=1}^{t} |A +_i B| \geq t \min\{p, |A| + |B| - t\}.$$

When $|G| = p$ and $t = 1$, both results coincide. It is an open question to give a Kneser-type generalization of Pollard's Theorem to a general abelian group G. The best partial result is a theorem that describes the structure of A and B when

$$\sum_{i=1}^{t} |A +_i B| < t|A| + t|B| - (2t^2 - 3t + 2),$$

showing there must exist $A' \subseteq A$ and $B' \subseteq B$ with $|A \setminus A'| + |B \setminus B'| \leq t - 1$, $A' +_t B = A' + B' = A +_t B$, and

$$\sum_{i=1}^{t} |A +_i B| \geq t(|A| + |B| - |H|),$$

where H is the stabilizer of $A' + B' = A +_t B$. These conclusions, combined with classical sumset results, imply a strong structural description of A and B. However, the term $2t^2 - 3t + 2$ is too large for this result to encompass a full generalization of both Kneser and Pollard's Theorems, which would require a result valid using t^2 rather than $2t^2 - 3t + 2$. Here we achieve progress by improving the main quadratic term in the bound from $2t^2$ to $\frac{4}{3}t^2$. Joint work with Runze Wang.

YIFAN JING, Ohio State Univerisity
Measure doubling for small sets in compact Lie groups

A central problem in additive combinatorics is to understand how the size of a sumset (or product set) compares to the size of the original set, and to describe the underlying structure when this "doubling" is small. In this talk, I will survey some classical results in the area and discuss recent developments in the setting of compact Lie groups, based on joint work with Chieu-Minh Tran and Simon Machado.

YU-RU LIU, U. of Waterloo
Equidistribution Theorems in Additive Combinatorics

We establish a function-field analogue of Weyl's equidistribution theorem for polynomial sequences and explore its applications to problems in additive combinatorics. This is joint work with Jérémy Champagne, Thái Hoàng Lê and Trevor Wooley.

ANTON MOSUNOV, Cornell University
Numbers that are integrally representable by the homogenization of the minimal polynomial of $\tan(\pi/n)$

Let $F(x, y)$ denote a binary form with integer coefficients of degree $d > 2$ and non-zero discriminant. Let $R_F(Z)$ denote the number of all integers $\leq Z$ that are integrally representable by F. In 2019, Stewart and Xiao proved that there exists a positive constant C, which depends only on F, such that $R_F(Z) \sim CZ^{2/d}$. For every integer $n > 2$, we estimate the constant C for $t_n(x, y)$, the homogenization of the minimal polynomial of $\tan(\pi/n)$.

STEVEN SENGER, Missouri State University
Gaps in popular iterated sumset sizes

We discuss the triangular gaps observed experimentally by Mel Nathanson and later by Kevin O'Bryant in the most popular sizes of the h-fold iterated sumset, hA, when A is a randomly chosen four-element subset of the first q natural numbers, for q much larger than h.

FERNANDO XUANCHENG SHAO, University of Kentucky
Recent developments on the polynomial Szemeredi theorem

As a special case of the celebrated theorem of Bergelson and Leibman (the polynomial Szemeredi theorem), any positive density subset of the integers must contain a polynomial progression of the form $x, x+y, x+y^2$ with y nonzero. In the last five years since the pioneering work of Peluse and Prendiville, there have been numerous developments on the quantitative aspects of such results. I will give a brief overview of these recent developments, before describing a two-dimensional version and a "popular" version of the polynomial Szemeredi theorem for the pattern $x, x+y, x+y^2$. The talk includes joint works with Sarah Peluse, Sean Prendiville, and Mengdi Wang.

HUNTER SPINK, University of Toronto
Geometric additive combinatorics via o-minimality

In this talk I will introduce (gently!) o-minimality as a tool for doing additive combinatorics in geometric settings, based on joint work with Jacob Fox and Matthew Kwan.

As an application, if we remove all line segments contained in a "nice" subset $M \subset \mathbb{R}^n$ (e.g. $M = \{e^{2x^2 - e^{log(xyz)log(x+yz)}/3x} - x^{x-y} <= 6\} \subset \mathbb{R}^3$), then the probability that a randomly signed sum of nonzero vectors $\sum \epsilon_i v_i$ lies in M is $n^{-\frac{1}{2}+o(1)}$, essentially matching the $O(n^{-1/2})$ bound from classical Littlewood–Offord theory for M a single point.

JONATHAN TIDOR, Princeton University
Uniform sets with few 4APs via colorings

A well-known question of Ruzsa asks if there exist Fourier-uniform sets with very few 4-term arithmetic progressions: do there exist Fourier-uniform subsets of $\mathbb{Z}/N\mathbb{Z}$ with density α and 4AP-density $\alpha^{\omega(1)}$? I will discuss a surprising connection between this problem and one in arithmetic Ramsey theory. We show that one could construct such sets given a coloring of $\{1, 2, \ldots, N\}$ with $N^{o(1)}$ colors that avoids symmetrically colored 4APs. We say that a 4AP is symmetrically colored if its first and last term receive the same color and its middle two terms receive the same color.

Based on joint work with Mingyang Deng and Yufei Zhao.

STANLEY YAO XIAO, UNBC
Primes of the form $f(p,q)$, f quadratic, and applications

We capitalize on the breakthrough result of Green and Sawhney proving the infinitude of primes of the form $p^2 + nq^2$, where $n \equiv 0, 4 \pmod 6$ is a fixed positive integer and p, q are prime variables to arbitrary binary quadratic forms satisfying the obvious non-degeneracy conditions. Notably, our result covers irreducible indefinite binary quadratic forms. This has applications to counting elliptic curves admitting a rational isogeny of prime degree, ordered by conductor.