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On the equivalence of c -potentiability and c -path boundedness in the sense of Artstein-Avidan, Sadovsky and Wyczesany.

A cornerstone of convex analysis, established by Rockafellar in 1966, asserts that a set has a potential if and only if it is cyclically monotone. This characterization was generalized to hold for any real-valued cost function c and lies at the core structure of optimal transport plans. However, this equivalence fails to hold for costs that attain infinite values. In this talk, we explore potentiability for an infinite-valued cost c under the assumption of c -path boundedness, a condition that was first introduced by Artstein-Avidan, Sadovsky and Wyczesany. This condition is necessary for potentiability and is more restrictive than c -cyclic monotonicity. We provide general settings and other conditions under which c -path boundedness is sufficient for potentiability, and therefore equivalent. We provide a general theorem for potentiability, requiring no topological assumptions on the spaces or the cost. We then provide sufficiency in separable metric spaces and costs that are continuous in their domain. Finally, we introduce the notion of a c -path bounded extension and use it to prove the existence of potentials for a special class of costs on $\mathbb{R}^2$. We illustrate our discussion and results with several examples.