MAHISHANKA WITHANACHCHI, University of Calgary

Complex Analytic Methods in One Dimensional Scattering: Harmonic Exponentials, Inner Functions, and Toeplitz Kernels

One dimensional scattering for Schrödinger type and impedance form operators admits a rich complex analytic structure. In recent work, the scattering matrix can be written explicitly in terms of a harmonic exponential associated with the impedance profile, yielding closed form expressions for reflection and transmission coefficients as bounded analytic functions on the upper half plane. This description fits naturally into the framework of Hardy spaces, inner and outer factorization, and $SU(1,1)$ transfer matrices.

In this talk, I will explain how the transmission coefficient appears as an outer function in $H^\infty(\mathbb{C}_+)$ and how this connects to classical tools of complex analysis, including Poisson integrals, boundary uniqueness, and phaseless inverse problems. I will then discuss how the product integral formulation of the transfer matrix can be interpreted in the language of de Branges spaces and related to the Makarov and Poltoratski theory of meromorphic inner functions and Toeplitz kernels for canonical systems. This perspective suggests new ways to study completeness, spectral synthesis, and inverse scattering at low regularity by analyzing Toeplitz operators with symbols built from scattering data.

If time permits, I will outline ongoing work that uses these complex analytic techniques to formulate inverse and rigidity results for layered and low regularity media, emphasizing the role of Hardy space methods and the structure of inner and outer factors arising from the scattering matrix.