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L-functions and Numerical Computations Concerning Landau-Siegel Zeros

Let χ be a Dirichlet character mod q, $L(s,\chi) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s}$ be the associated Dirichlet L-function, and $s = \sigma + it$ be a complex number. (If $\chi \equiv 1$ then L is the Riemann zeta function.) Let $\zeta_q(s) = \prod_{\chi} L(s,\chi)$ where χ ranges over all Dirichlet characters of modulus q. A theorem due to Landau showed that there is a constant $c > 0$ such that for all q, $\zeta_q(s)$ has at most one zero in the region

$$\sigma \geq 1 - \frac{c}{\log\left(q(|t|+1)\right)}, \tag{1}$$

and furthermore if there is such a zero, then it is necessarily real and the associated character χ is quadratic. Such a zero, if it exists, is called a Landau-Siegel zero or an exceptional zero. It is a particular kind of counterexample to the Generalized Riemann Hypothesis. Proving the nonexistence of Landau-Siegel zeros is of great interest, with applications to e.g. bounds on class numbers of quadratic number fields.

In this talk, we will demonstrate a new computational result regarding the non-existence of such zeros. Following methods that were developed by Heath-Brown, and Thorner and Zaman, we refined an inequality concerning the logarithmic derivative of Dirichlet L-functions and their largest real zeros. With modern computing clusters, we utilized this inequality and computationally verified that Landau-Siegel zeros do not exist for any quadratic character of modulus $q \leq 10^{10}$, with $c = 1/5$ in the above region.