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On the length of cyclic algebras

Cyclic algebras were first introduced by Leonard Eugene Dickson in 1906. He remarked that, for appropriate choice of parameters, his construction yielded a division algebra. These cyclic division algebras were one of the earliest examples of non-commutative division algebras over a field and are now applied in wireless communication systems (e.g. 4G LTE, 5G, Wi-Fi networks) via space-time block coding.

A notable numerical invariant of any algebra over a field is its length, defined as the length of its longest chain of linear subspaces. In 2016, the length of 4-dimensional cyclic algebras was proven to be 2. Five years later, the length of cyclic algebras of dimension 9, 16, and 25 was stated to be 4, 6, and 8 respectively. We show these latter values are ill-derived, meaning the length of cyclic algebras of dimension greater than 4 is still an open problem.

In pursuit of solving this open problem, we construct an infinite family of cyclic division algebras and give a lower bound on the length of its members. Lastly, we tensor members of a subfamily with the Gaussian rationals to produce fully-diverse linear space-time block codes with non-vanishing determinant.